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# Correlations between Saturation Properties of Isospin Symmetric and Asymmetric Nuclear Matter in a Nonlinear $\sigma$-$\omega$-$\rho$ Mean-field Approximation

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**Abstract**

The density-dependent correlations and Fermi-liquid properties of isospin-symmetric and asymmetric nuclear matter are discussed in terms of density-dependent effective masses and effective coupling constants in a nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation with self-interactions and mixed-interactions of mesons. The nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation is constructed with conditions of conserving approximations which generate *effective masses* ($M_N^*$, $m_\sigma^*$, $m_\omega^*$, $m_\rho^*$) and *effective coupling constants* ($g_\sigma^*$, $g_\omega^*$, $g_\rho^*$) of baryons and mesons. The current nonlinear mean-field approximation becomes a thermodynamically consistent approximation that maintains Hugenholtz-Van Hove theorem and Landau's requirement of quasiparticles. The current nonlinear mean-field approximation is applied to isospin-asymmetric, neutron-rich nuclear matter which is defined as beta-equilibrium nuclear matter. The density-dependent correlations among binding energies and densities, Fermi-liquid properties of isospin-symmetric $(n, p)$ nuclear matter and isospin-asymmetric, neutron-rich $(n, p, e)$ nuclear matter are discussed. The saturation properties of isospin-asymmetric, neutron-rich nuclear matter of magic nuclei are calculated. The analysis suggests that a neutron star be a giant nuclear furnace to produce neutron-rich, heavy nuclei on the surface of the star.

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# 1 Introduction

The binding energy of symmetric nuclear matter exhibits a saturation point which has an energy per particle between $-15$ MeV and $-17$ MeV at a Fermi-momentum $k_F$ in the range from 1.29 fm$^{-1}$ to 1.44 fm$^{-1}$. The empirical fact indicates that any assumed two-body potential must produce the experimental data if a potential is to account for the properties of nuclei, and hence, one can either accept or reject the potential by calculating whether the binding energy and density can be reproduced or not. The result of traditional nuclear matter calculations shows that it produces saturation at too high a density[1], although a realistic two-body potential yields the correct binding energy per particle. In order to resolve the problem, so many models and approximations, listing a few, such as, three-body interactions[2], effective many-body approximations[3], the reduced density-operator method[4], the real-time formulation of nonequilibrium field theory[5, 6], relativistic nuclear many-body models[7]–[10], and quark-induced effective models[11], have been applied; the problem of nuclear matter saturation has been known as one of the fundamental and complex problems in nuclear many-body physics. The investigations of the saturation problem have conjoined properties of the theory of many-body physics with self-consistency to approximations of strongly interacting particles. Besides the saturation problem, there are several empirical quantities calculated at saturation density, such as compressibility, $K$, and symmetry energy $a_4$, which are expected experimentally to be 200 MeV $\sim$ 300 MeV, and 30 MeV $\sim$ 40 MeV, respectively. The interactions of particles concerning these observables and binding energy at saturation have been discussed in terms of self-consistency and density-dependent correlations in the theory of many-body problems.

Although hadrons interact strongly, the independent particle picture of hadrons has been studied[12] in nuclear physics. Nuclear many-body approximations applied to infinite nuclear and neutron matter are required to maintain self-consistent conditions: the Hugenholtz-Van Hove theorem[13] (HV theorem), the virial theorem[14], the theory of Landau's quasiparticles[15, 16], and the theory of conserving approximations[17]–[19]. The self-consistent conditions are essential to calculate microscopic and macroscopic physical quantities explicitly and denoted as *thermodynamic consistency*[20]–[23] to an approximation. A relativistic quantum field theory model containing baryons and neutral scalar and vector mesons was proposed by Walecka to study bulk properties of high density matter[7]. The linear $\sigma$-$\omega$ mean-field approximation has been extensively applied to finite nuclei, nucleon-nucleon interactions as well as to equations of state of nuclear matter and properties of neutron stars[24, 21]. The structure of the linear mean-field approximation is extended to nonlinear mean-field models which have also been extensively applied to properties of nuclear and neutron matter[25, 26]. Neutron stars have attracted much interest in nuclear and high energy physics with a large body of accumulating experimental evidence. The observed masses of hadronic neutron stars are above 1.3 $M_\odot$ (the solar mass: $M_\odot \sim 1.989 \times 10^{30}$ kg), and the maximum masses of neutron stars are expected to be below 2.5 $M_\odot$[27], or $2.2 \pm 0.2 M_\odot$[28]. The analyses of nuclear matter and neutron stars have revealed that the microscopic and macroscopic observables are self-consistently related to each other by conditions of thermodynamic consistency. Many-body interactions and many-body effects, such as retardation effects and density-dependence, are essential to determine observables of finite nuclei, nuclear matter and astrophysical objects[29]–[31].

Fermi-liquid properties at saturation are strongly correlated through density-dependence with properties in high densities, such as the mass and radius of neutron stars. The density-dependence is essentially related with self-consistency and conditions of thermodynamic consistency, and it is again provoking attentions in the analysis of thermodynamic consistency to microscopic and macroscopic dynamics and properties of nuclear matter and neutron stars. In the current investigation, the relativistic field theory model by Serot and Walecka[7, 24] is employed and extended to the nonlinear $\sigma$-$\omega$-$\rho$ mean-field model by maintaining thermodynamic consistency[23], which leads to the concept of *effective masses of mesons* and *effective coupling constants* produced by nonlinear interactions of hadrons. The effective masses and coupling constants are mutually interrelated with self-consistency; therefore, one must be careful when one will introduce effective interactions to an approximation. The mean-field approximation with nonlinear interactions of baryons and mesons will become a conserving approximation when effective masses of mesons and effective vertex coupling constants are self-consistently defined[23]. The conditions of thermodynamic consistency to nonlinear interactions of hadrons can explicitly generate density-dependent, or energy-dependent effective masses and effective coupling constants.

The nonlinear mean-field model contains several adjustable parameters coming from self-interactions and mixed-interactions of mesons and baryons. However, the nonlinear coupling constants will be adjusted and fixed from the outset in order to reproduce saturation properties of isospin symmetric $(n, p)$ nuclear matter whose binding energy is $-15.75$ MeV at $k_F = 1.30$ fm$^{-1}$, symmetry energy, $a_4 = 30.0$ MeV[23], as well as the maximum masses of neutron stars, $M_{max} = 2.50$ $M_\odot$ or 2.00 $M_\odot$ for high density regions. Since nonlinear coupling constants are self-consistently interrelated by way of equations of motion and conditions of thermodynamic consistency[18, 23], effective masses and coupling constants become intensely density-dependent and are confined in certain values so as to reproduce observables of low and high densities; in other words, coupling constants of nonlinear interactions are not free parameters to adjust at all. This is an important consequence derived from the conserving nonlinear mean-field approximation[23].

Macroscopic quantities, such as energy density, pressure, entropy density and particle density, have to satisfy thermodynamic relations, and also, those macroscopic quantities are constrained microscopically by way of energy-momentum conservations; that is, the microscopic and macroscopic quantities are self-consistently interrelated to each other. This is the self-consistency of many-body problems, which has been discussed and clarified as the requirement that the propagators and self-energies have to satisfy the '$\Phi$-derivable'-condition [17, 18, 23]. The microscopic and macroscopic requirements are denoted as *thermodynamic consistency*[20, 22], and the approximations which maintain thermodynamic consistency are called *conserving approximations.* In articles, the consistency of Green functions, self-energies and energy density is not often examined clearly; the self-consistent conditions for the propagators and self-energies are not automatically maintained even in the level of nonlinear (Hartree) mean-field approximations. The current nonlinear mean-field approximation is constructed to maintain thermodynamic consistency, which constrains nonlinear interactions and adjustable coefficients in terms of self-consistency and physical conservation laws. It should clarify physical meanings of nonlinear interactions in nonlinear mean-field approximations and exhibit model-independent results for nuclear many-body physics. The self-consistency and thermodynamic consistency will be explained in the sec. 3.

The treatment of quantum vacuum is typically neglected in hadronic models and complicated many-body interactions and mechanisms generate retardation effects and strong density-dependent correlations. Because of these fundamental phenomena, it may be difficult to study nuclear systems at short distances or high-energy regions[24, 21]. However, the general features obtained in the calculation may compensate for the deficiency and remain valid as model-independent results, since the current nonlinear mean-field approximation constructed with thermodynamic consistency is a self-consistent approximation which constrains nonlinear interactions and adjustable parameters in terms of physical conservation laws. Self-consistency and thermodynamic consistency are essential in the nonperturbative approach, and by checking self-consistent, density-dependent correlations of observables for low and high energy, it may be possible to investigate and understand validities of nuclear models.

The nonlinear mean-field interactions exhibit significant density-dependent effects on properties of nuclear and neutron matter[23]. Although nonlinear coefficients can be introduced as free parameters at the outset, nonlinear interactions will be confined by self-consistency; that is, the values of nonlinear coefficients are confined and their contributions to physical quantities are obtained in a balanced and restricted way. Therefore, it is important to examine the existence and strength of nonlinear interactions of hadrons, whether effective masses of mesons are measured from the nuclear experimental data or not. The renormalized effective masses and coupling constants are derived explicitly, and density-dependent contributions of nonlinear interactions to properties of nuclear and neutron matter are quantitatively checked. The current nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation which reproduces saturation properties and the maximum mass of neutron stars is extended to isospin asymmetric matter and applied to, for example, neutron-rich nuclear matter defined by way of beta-equilibrium $(n, p, e)$ matter. It is observed that the *isospin-asymmetry and density-dependence* are both essential to calculate the binding energy, symmetry energy and compressibility of stable neutron-rich nuclear matter for low density regions. The isospin-asymmetric, charge-neutral nuclear matter does not produce the saturation of binding energy for low density regions in the nonlinear mean-field (Hartree) approximation. The isospin-asymmetric nuclear matter without the electric charge and charge-neutrality can exhibit the saturation properties of binding energy. The nonlinear mean-field approximation is applied to study saturation properties, the binding energy and density, compressibility and symmetry energy, of neutron-rich nuclear matter, which should be used to study properties of neutron-rich magic nuclei. The properties of neutron-rich nuclear matter corresponding to magic nuclei as well as symmetric nuclei will be important constraints for models of nuclear physics. Therefore, we suggest that experimental values of Fermi-liquid properties of neutron-rich nuclear matter corresponding to the stable magic nuclei should be investigated quantitatively to understand models of nuclear physics. The saturation properties of neutron-rich nuclear matter in the nonlinear mean-field approximation are explained in the sec. 6.

The properties of neutron stars, such as the mass, moment of inertia and radius of neutron stars, are calculated by employing the equation of state in the nonlinear mean-field approximation. The coupling constants are fixed by reproducing saturation properties and the symmetry energy, $a_4 = 30.0$ MeV of $(n, p)$ symmetric nuclear matter and the maximum masses of neutron stars. We have calculated properties of nuclear matter and neutron stars (nuclear compressibility, symmetry energy, effective masses of nucleons and mesons, the maximum mass of neutron stars) and investigated the minimum value of nuclear compressibility, $K$, to the value of the supposed maximum mass of neutron stars. The relativistic effects, density-dependence and isospin-asymmetry to symmetry energy[32]–[37] are clearly examined in the current mean-field approximation. The equation of state for isospin-asymmetric nuclear matter becomes softer than that of pure-neutron matter, which requires a larger compressibility in order to support the maximum mass of neutron stars. For example, the equation of state for pure-neutron stars requires that a lower bound of compressibility $K > 160$ MeV be necessary to support the maximum mass, $M_{max}(n) > 2.00\ M_{\odot}$, whereas in the case of isospin-asymmetric neutron stars, $K > 250$ MeV be necessary to support $M_{max}(n,p) > 2.00\ M_{\odot}$ [23].

The density-dependent correlations of compressibility and symmetry energy, effective masses and coupling constants in the nonlinear mean-field approximation are discussed and shown in the sec. 5 and sec. 6. It is difficult to reproduce empirical values of $K$ and $a_4$, $M_{max}$, simultaneously by adjusting the values of coupling constants, since the conditions of thermodynamic consistency and coupled equations of motion for mesons and baryons strictly restrict the values of nonlinear coefficients. Consequently, the effective coupling constants and effective masses of hadrons are confined. The nonlinear interactions will yield certain fixed values of observables which should be checked by experimental data. The strong density-dependent correlations between properties of nuclear matter and isospin-asymmetry have exhibited the saturation of symmetry energy in high densities; the nonlinear interactions of mesons and isospin-asymmetry are necessary and sufficient to generate the saturation of symmetry energy[38]. This is one of the important results derived in the nonlinear mean-field approximation.

As the density-dependent correlations between isospin-symmetric nuclear matter and neutron stars have been investigated, the correlations between properties of isospin-symmetric and asymmetric nuclear matter will be examined with the current nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation. The applications of the self-consistent mean-field approximation upon $(n,p)$ isospin-symmetric and isospin-asymmetric nuclear matter and the analysis of effective masses and coupling constants, binding energies and saturation properties are the purpose of the current paper. The strong density-dependence of properties of nuclear matter, self-consistency of meson-propagators with effective masses of mesons and effective coupling constants, the structure of neutron stars may be clarified. Since the effective masses and coupling constants are intricately density-dependent, these density-dependent correlations may also be examined in properties of isospin asymmetric $(n,p,e)$ nuclear matter corresponding to stable magic nuclei, such as Ba $(Z,A,N = 56,138,82)$ and Pb $(Z,A,N = 82,208,126)$. Conclusions and remarks on the self-consistent nonlinear mean-field approximation are discussed in the sec. 7.

## 2 The nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation with density-dependent effective coupling constants ($g^*_\sigma$, $g^*_\omega$, $g^*_\rho$)

The nonlinear $\sigma$-$\omega$-$\rho$ mean-field lagrangian with self- and mixed-interactions of mesons is givey by[23],

$$\begin{aligned}\mathcal{L}_{NHA} = & \sum_N \bar{\psi}_N \Big[ i\gamma_\mu \partial^\mu - g_\omega \gamma_0 V_0 - \frac{g_\rho}{2}\gamma_0 \tau_3 R_0 - (M_N - g_\sigma \phi_0)\Big]\psi_N \\ & - \frac{1}{2} m_\sigma^2 \phi_0^2 - \frac{g_{\sigma 3}}{3!}\phi_0^3 - \frac{g_{\sigma 4}}{4!}\phi_0^4 + \frac{1}{2} m_\omega^2 V_0^2 + \frac{g_{\omega 4}}{4!} V_0^4 + \frac{g_{\sigma\omega}}{4}\phi_0^2 V_0^2 \\ & + \frac{1}{2} m_\rho^2 R_0^2 + \frac{g_{\rho 4}}{4!} R_0^4 + \frac{g_{\sigma\rho}}{4}\phi_0^2 R_0^2 + \frac{g_{\omega\rho}}{4} V_0^2 R_0^2 \ ,\end{aligned} \tag{2.1}$$

where $\psi_N$ $(N = n, p)$ is the field of nucleon; the meson-fields operators are replaced by expectation values in the ground state: $\phi_0$ for the $\sigma$-field, $V_0$ for the vector-isoscalar $\omega$-meson, $V_\mu V^\mu = V_0^2 - \mathbf{V}^2$, $(\mu = 0, 1, 2, 3)$. The neutral $\rho$-meson mean-field, $R_0$, is chosen for $\tau_3$-direction in isospin space. The current nonlinear mean-field approximation will be constructed so that the framework of Serot and Walecka's linear $\sigma$-$\omega$ mean-field approximation[24] is maintained, such as Lorentz-invariance and renormalizability, thermodynamic consistency: the Hugenholtz-Van Hove theorem[13], the virial theorem[14], and the theory of conserving approximations[18, 23]. The masses in (2.1) are fixed as $M_N = 939$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV and $m_\rho = 770$ MeV, in order to compare the effects of nonlinear interactions with those of the linear $\sigma$-$\omega$ approximation discussed by Serot and Walecka.

Nonlinear mean-field interactions of baryons and mesons can be classified as mass-renormalizable, vertex-renormalizable and source-renormalizable terms[23], and the self-interactions and mixed-interactions of mesons in the lagrangian $\mathcal{L}_{NHA}$ can be renormalized as effective masses of mesons. The renormalized quantities are important to examine density-dependence of observables, the conditions of thermodynamic consistency; the theory of conserving approximations will provide ideas and methods to introduce renormalized effective masses, vertices and sources. The lagrangian (2.1) produces *effective masses* of hadrons, but not *effective coupling constants*. The concept of effective coupling constants is introduced based on Serot and Walecka's mean-field approximation, nonperturbative propagators of baryons and mesons and self-consistent conditions of thermodynamic consistency[18, 23]. The nonlinear coupling coefficients are defined as $\sigma\sigma N$, $\sigma\omega N$ and $\sigma\rho N$ nonlinear vertex interactions. The density-dependent scalar, vector and vector-isovector coupling constants, $g_\sigma^*$, $g_\omega^*$, $g_\rho^*$, will be generated by employing the scalar mean-field, $\phi_0$, so that the Lorentz-covariant, renormalizable structure of Serot and Walecka's approximation scheme[24] should be maintained. We denote the mean-field model of the density-dependent effective masses and vertex-interactions as $\mathcal{L}'_{NHA}$, and it will be assumed in the form[38],

$$\mathcal{L}'_{NHA} = \mathcal{L}_{NHA} + \bar{\psi}_N \Big[ -g_{\sigma\omega N}\phi_0 \gamma_0 V_0 - g_{\sigma\rho N}\phi_0 \gamma_0 \tau_3 R_0 + \frac{g_{\sigma\sigma N}}{2}\phi_0^2 \Big]\psi_N \ , \tag{2.2}$$

and the Green's function for the Fermi-sea particle approach in the rest frame of nuclear matter is used:

$$G_D(k) = (\gamma^\mu k^*_\mu + M^*_N(k)) \frac{i\pi}{E^*(k)} \delta(k^0 - E(k)) \theta(k_F - |\mathbf{k}|) \ , \tag{2.3}$$

where $k^{*0} = k^0 + \Sigma^0$. The effective nucleon mass is $M^*_N(k_F) = M_N - g^*_\sigma \phi_0 = M_N + \Sigma^s(k_F)$, and the baryon single particle energy is $E(k) = E^*(k) - \Sigma^0(k)$, with $E^*(k) = (\mathbf{k}^2 + M^{*2}_N)^{1/2}$. The $\omega$-meson contribution to the baryon self-energy is given by $\Sigma^0_\omega = -g^*_\omega V_0$, and $\rho$-meson contributions to the self-energies for proton and neutron are denoted as $\Sigma^0_{\rho p} = -(g^*_\rho/2)R_0$ and $\Sigma^0_{\rho n} = (g^*_\rho/2)R_0$, respectively.

Nonlinear mean-field interactions of baryons and mesons can be classified as mass-renormalizable, vertex-renormalizable and source-renormalizable terms[23]. We are interested in density-dependent correlations of infinite matter induced by renormalized vertex interactions and sources. Since the renormalized quantities are important to examine density-dependence of observables, one should be careful to introduce effective masses, vertices and sources. The lagrangian (2.1) produces *effective masses* of hadrons, but not *effective coupling constants*, whereas the lagrangian (2.2) produces both *effective masses* and *effective coupling constants.* The nonlinear cubic interactions such as $\phi_0 V_0^2$ and $\phi_0 R_0^2$ are not considered in the current approximation, since these terms do not produce noticeable contributions to properties of infinite matter compared to $g_\sigma$, $g_\omega$, $g_{\sigma\omega}$, $g_{\sigma 4}$, $g_{\omega 4}$ terms, although nonlinear cubic terms may work as sensitive adjustments in reproducing experimental values of finite nuclear system.

The nonperturbative $\sigma$, $\omega$ and $\rho$ meson-propagators $D^{(0)}_\sigma(k)$, $D^{(0)\mu\nu}_\omega(k)$, and $D^{(0)\mu\nu}_{\rho\ ab}(k)$ for the nonlinear mean-field approximation are defined as:

$$D^{(0)}_\sigma(k) = \frac{1}{k^2_\lambda - m^{*2}_\sigma + i\varepsilon} \ , \qquad D^{(0)\mu\nu}_\omega(k) = \frac{-g^{\mu\nu}}{k^2_\lambda - m^{*2}_\omega + i\varepsilon} \ , \tag{2.4}$$

and

$$D^{(0)\mu\nu}_{\rho\ ab}(k) = \frac{-g^{\mu\nu}\delta_{ab}}{k^2_\lambda - m^{*2}_\rho + i\varepsilon} \ , \tag{2.5}$$

where $m^*_\sigma$, $m^*_\omega$ and $m^*_\rho$ are effective masses of $\sigma$, $\omega$, $\rho$ mesons, respectively. The effective masses are density-dependent constants given by nonlinear interactions of mesons and baryons, and therefore, they have to be determined simultaneously when energy density, pressure and self-energies are self-consistently determined. The equation of motion for baryons is given by,

$$\Big[(i\gamma_\mu \partial^\mu - g^*_\omega \gamma_0 V_0 - \frac{g^*_\rho}{2}\gamma_0 \tau_3 R_0) - (M - g^*_\sigma \phi_0)\Big]\psi_N = 0 \ , \tag{2.6}$$

where $g^*_\sigma$, $g^*_\omega$ and $g^*_\rho$ are *effective coupling constants* for $\sigma$, $\omega$ and $\rho$ mesons. They are defined as,

$$\begin{aligned} g_\sigma^* =& g_\sigma + (g_{\sigma\sigma N}/2)\phi_0 \ , \\ g_\omega^* =& g_\omega + g_{\sigma\omega N}\phi_0 \ , \\ g_\rho^*/2 =& g_\rho/2 + g_{\sigma\rho N}\phi_0 \ . \end{aligned} \tag{2.7}$$

The equations of motion for mesons in the mean-field lagrangian (2.2) are given by,

$$\begin{aligned} & m_\sigma^2\phi_0 + \frac{g_{\sigma 3}}{2!}\phi_0^2 + \frac{g_{\sigma 4}}{3!}\phi_0^3 - \frac{g_{\sigma\omega}}{2}V_0^2\phi_0 - \frac{g_{\sigma\rho}}{2}R_0^2\phi_0 - \frac{g_{\sigma\sigma N}}{2}\rho_s\phi_0 \\ & = g_\sigma^*\rho_s - g_{\sigma\omega N}V_0\rho_B - g_{\sigma\rho N}R_0\rho_3 \ , \\ & m_\omega^2 V_0 + \frac{g_{\omega 4}}{3!}V_0^3 + \frac{g_{\sigma\omega}}{2}\phi_0^2 V_0 + \frac{g_{\omega\rho}}{2}R_0^2 V_0 = g_\omega^*\rho_B \ , \\ & m_\rho^2 R_0 + \frac{g_{\rho 4}}{3!}R_0^3 + \frac{g_{\sigma\rho}}{2}\phi_0^2 R_0 + \frac{g_{\omega\rho}}{2}V_0^2 R_0 = \frac{g_\rho^*}{2}\rho_3 \ . \end{aligned} \tag{2.8}$$

Note that the equations of motion for $\sigma$-field acquire a mass term, $\frac{g_{\sigma\sigma N}}{2}\rho_s\phi_0$, and new source terms, $-g_{\sigma\omega N}V_0\rho_B - g_{\sigma\rho N}R_0\rho_3$ from nonlinear vertex interactions. The equations of motion for $\sigma$-field is intensely modified since the effective vertex interactions are defined with the scalar mean-field, $\phi_0$, so that the Lorentz-covariance of the nonlinear mean-field model is maintained. The effective masses and coupling constants are self-consistently connected to one another, and the density-dependence by way of masses, nonlinear vertex interactions and sources are explicitly seen in the eq. (2.8). The equations of motion for $\omega$ and $\rho$ mesons are formally equivalent to those of the linear mean-field approximation (LHA), if coupling constants of LHA, $g_\omega$ and $g_\rho$, are replaced by $g_\omega^*$ and $g_\rho^*$.

The self-energies in the nonlinear approximation are calculated by the Feynman diagram in Fig. 1 as,

$$\Sigma^s = i\frac{g_\sigma^*}{m_\sigma^{*2}}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\Big\{(g_\sigma^* - g_{\sigma\omega N}V_0\gamma^0 - g_{\sigma\rho N}R_0\gamma^0\tau_3)G_D(q)\Big\} = -\frac{g_\sigma^{*2}}{m_\sigma^{*2}}\rho_s^* \ , \tag{2.9}$$

where $\rho_s^*$ is the modified scalar density defined by $g_\sigma^*\rho_s^* = g_\sigma^*\rho_s - g_{\sigma\omega N}V_0\rho_B - g_{\sigma\rho N}R_0\rho_3$. Note that the scalar self-energy, $\Sigma^s$, is generated by three density sources in the current nonlinear model. The $\omega$-meson and $\rho$-meson self-energies are given by

$$\Sigma_\omega^\mu = i\frac{g_\omega^{*2}}{m_\omega^{*2}}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\Big(\gamma^\mu G_D(q)\Big) = -\frac{g_\omega^{*2}}{m_\omega^{*2}}\rho_B\delta_{\mu,0} \ , \tag{2.10}$$

and

$$\Sigma_{\rho\binom{p}{n}}^\mu = \pm i\frac{g_\rho^{*2}}{4m_\rho^{*2}}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\Big(\tau_3\gamma^\mu G_D(q)\Big) = \mp\frac{g_\rho^{*2}}{4m_\rho^{*2}}\rho_3\delta_{\mu,0} \ , \tag{2.11}$$

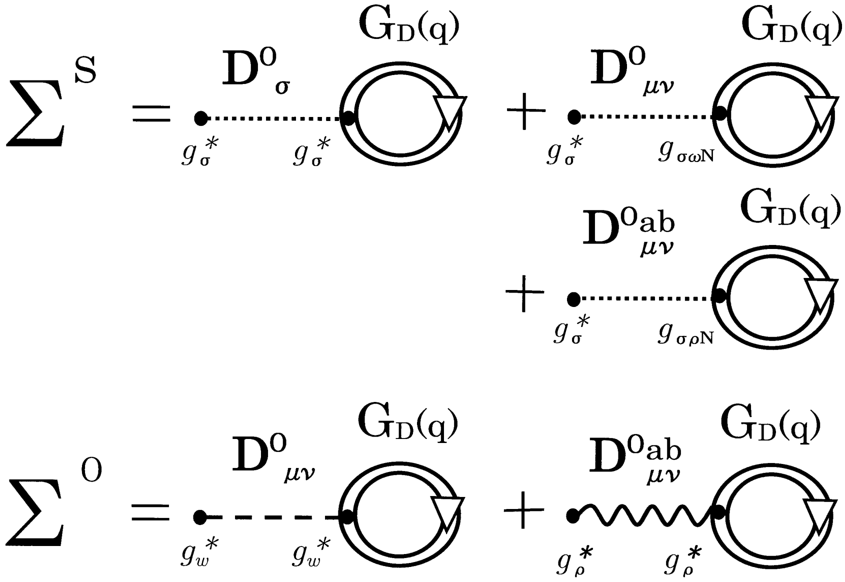


Fig. 1. The self-energies, $\Sigma^s$ and $\Sigma^0$, with effective coupling constants. The interaction lines and coupling constants are self-consistently defined by effective masses and effective coupling constants. The vertex factors are: $ig^*_\sigma$ for $\sigma N$, $-ig^*_\omega\gamma^\mu$ for $\omega N$ and $-i(g^*_\rho/2)\tau_3\gamma^\mu$ for $\rho N$ couplings. The vertex factors of scalar-source terms induced by $\phi$-field couplings are $-ig_{\sigma\omega N}V_0\gamma^0$ for $\sigma\omega N$-vertex and $-ig_{\sigma\rho N}R_0\tau_3\gamma^0$ for $\sigma\rho N$-vertex.

where $\tau_3$ is the isospin matrix. The effective masses thermodynamically consistent with the effective coupling constants (2.7) are required to be:

$$\begin{aligned} m^{*2}_\sigma &= m^2_\sigma\Big(1+\frac{g_{\sigma 3}}{2m^2_\sigma}\phi_0+\frac{g_{\sigma 4}}{3!m^2_\sigma}\phi_0^2-\frac{g_{\sigma\omega}}{2m^2_\sigma}V_0^2-\frac{g_{\sigma\rho}}{2m^2_\sigma}R_0^2-\frac{g_{\sigma\sigma N}}{2}\rho_s\Big)\ , \\ m^{*2}_\omega &= m^2_\omega\Big(1+\frac{g_{\omega 4}}{3!m^2_\omega}V_0^2+\frac{g_{\sigma\omega}}{2m^2_\omega}\phi_0^2+\frac{g_{\omega\rho}}{2m^2_\omega}R_0^2\Big)\ , \\ m^{*2}_\rho &= m^2_\rho\Big(1+\frac{g_{\rho 4}}{3!m^2_\rho}R_0^2+\frac{g_{\sigma\rho}}{2m^2_\rho}\phi_0^2+\frac{g_{\omega\rho}}{2m^2_\rho}V_0^2\Big)\ . \end{aligned} \tag{2.12}$$

Note that the nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation is thermodynamically consistent only if effective masses and coupling constants are renormalized as (2.7) and (2.12). The effective coupling constants are as important as effective masses of hadrons in order to determine properties of isospin symmetric and asymmetric nuclear matter and neutron stars. The equations of motion and self-energies become formally equivalent to those of the linear mean-field approximation (LHA), if masses and coupling constants of mesons in LHA, $m_\sigma$, $m_\omega$, $m_\rho$ and $g_\sigma$, $g_\omega$, $g_\rho$, are replaced by $m^*_\sigma$, $m^*_\omega$, $m^*_\rho$ and $g^*_\sigma$, $g^*_\omega$, $g^*_\rho$. The effective coupling constants and effective masses of mesons and nucleon at saturation density of isospin-symmetric $(n, p)$ nuclear matter and properties of neutron

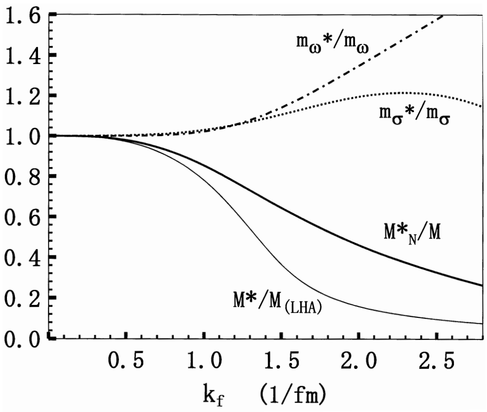


Fig. 2. (a) Effective masses of $(n, p)$ symmetric nuclear matter (NHA$^{2.50}$).

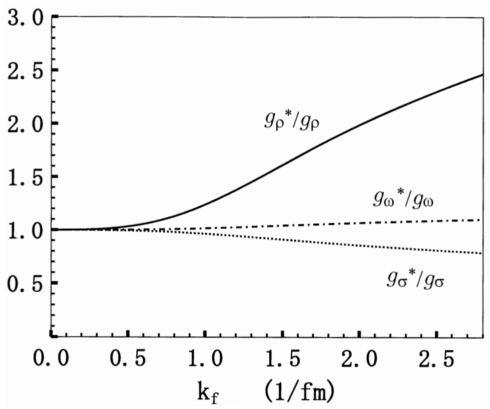


Fig. 2. (b) Effective coupling constants for $(n, p)$ symmetric nuclear matter (NHA$^{2.50}$).

stars are listed in the Table 1.

The coupling constants, $g_{\sigma\sigma N}$, $g_{\sigma\omega N}$ and $g_{\sigma\rho N}$, are determined by the imposed constraints: binding energy and searching for the lower bound of compressibility, $K$, with $a_4 = 30$ MeV and the maximum mass of neutron stars, $M_{max}$. The motivation of introducing the density-dependent coupling constants comes from the analyses of preceding nonlinear mean-field models whose nonlinear interactions are in principle renormalized as effective masses of mesons. As discussed in the ref. [23], nonlinear interactions in general produce mass-renormalizable, vertex-renormalizable and source-renormalizable terms required by conditions of thermodynamic consistency. Although one may expect that three more degrees of freedom ($g^*_\sigma$, $g^*_\omega$, $g^*_\rho$) seem to be in hand, solutions to the properties of infinite matter are restricted by the imposed constraints; consequently, the correlations between the upper bounds of effective masses and lower bound of compressibility are obtained. The coupling constants should be understood as upper bound values to get self-consistent solutions; for example, NHA$^{2.00}$ in the Table 2, $|g_{\sigma\sigma N}| \leq 0.070$, $|g_{\sigma\omega N}| \leq 0.020$ and $|g_{\sigma\rho N}| \leq 0.054$ are required to obtain solutions to the imposed constraints.

The energy density and pressure are calculated by way of energy-momentum tensor, and the equations of motion, self-energies, effective masses and coupling constants, ($M^*_N$, $m^*_\sigma$, $m^*_\omega$, $m^*_\rho$) and ($g^*_\sigma$, $g^*_\omega$, $g^*_\rho$), are self-consistently solved under the constraints of thermodynamic consistency. The effective masses and coupling constants in symmetric $(n, p)$ nuclear matter are shown in the Fig. 2 (a), (b) and Fig. 3 (a), (b), respectively. Notice that the effective mass of nucleons is given by $M^*_N = M_N - g_\sigma \phi_0$; in the linear $\sigma$-$\omega$ mean-field approximation (LHA), the sigma mean-field, $\phi_0$, increases rapidly, resulting in the rapid decrease of $M^*_N$[24], but in the nonlinear $\sigma$-$\omega$-$\rho$ approximation, $M^*_N$ is smoothed

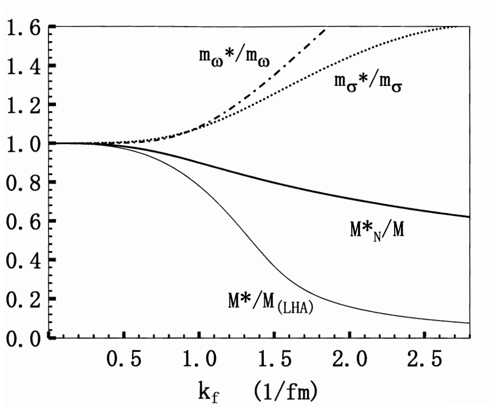


Fig. 3. (a) Effective masses of $(n, p)$ symmetric nuclear matter (NHA$^{2.00}$).

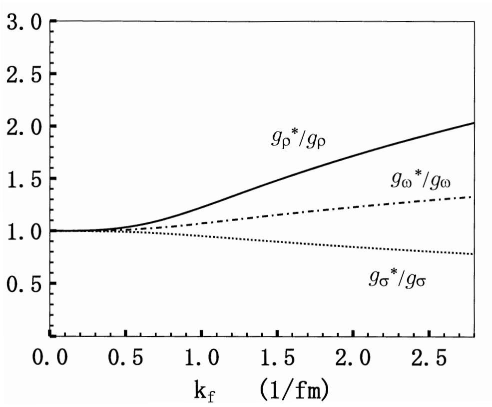


Fig. 3. (b) Effective coupling constants for $(n, p)$ symmetric nuclear matter (NHA$^{2.00}$).

and increased, $M_N^* = 0.7 \sim 0.8$, because the mean-field $\phi_0$ is suppressed by nonlinear interactions and self-consistency, which result in the increase of effective masses of $m_\sigma^*$ and $m_\omega^*$. The effective masses of $\sigma$ and $\omega$ mesons increase with density and become larger when the maximum mass of neutron stars are lowered as $M_{max}(n) = 2.50 \ \rightarrow 2.00 \ M_\odot$ in the numerical calculation. This is because the nonlinear interactions and density-dependences of effective masses of mesons cause to weaken repulsive forces in high densities, resulting in softer equations of state which produce small masses of neutron stars. Therefore, the smaller the neutron star mass, the greater the strength of nonlinear interactions and density-dependences are required. This is the consequence of density-dependent interactions (or retardation effects), which is explicitly confirmed in the current conserving mean-field approximation of hadrons.

The $\sigma$ and $\omega$ effective coupling constants are changed as, $g_\sigma^*/g_\sigma = 0.94 \sim 0.92$, $g_\omega^*/g_\omega = 1.03 \sim 1.12$ at saturation density, when the empirical data are altered from NHA$^{2.50}$ to NHA$^{2.00}$; the effective coupling constant changes $g_\rho^*/g_\rho = 1.38 \sim 1.45$ at saturation density (Table 1). The large values of $g_\rho^*/g_\rho$ are needed in order to reproduce the value of symmetry energy, $a_4 = 30.0$ MeV, in the nonlinear mean-field approximation[23, 38]. The important contributions to the symmetry energy also come from the exchange interactions[20], which will contribute to the value of the symmetry energy and may suppress the large values of $g_\rho^*/g_\rho$ at saturation density. Although the effective coupling constants and effective masses of mesons increase a little at saturation, they affect saturation properties of nuclear matter significantly, whereas those of neutron stars are mainly influenced by effective masses and nonlinear self-interactions in the mean-field approximation.

# 3 Thermodynamic Consistency of the current nonlinear mean-field approximation

Thermodynamic consistency requires consistent relations among equations of motion with self-energies, energy density and pressure, single-particle energy and propagators; Hugenholtz-Van Hove theorem and Landau's hypothesis of quasiparticles are acquired as the consequence of the theory of conserving approximations. The proof of thermodynamic consistency to an approximation is not only helpful to understand the structure of the approximation, but also essential for the analysis of Fermi-liquid properties and density-dependent

Table 1
Properties of symmetric nuclear matter and neutron stars with effective masses and effective coupling constants: ($M_N^*$, $m_\sigma^*$, $m_\omega^*$, $m_\rho^*$) and ($g_\sigma^*$, $g_\omega^*$, $g_\rho^*$).

| | $g_\sigma$ | $g_\omega$ | $g_\rho$ | $g_{\sigma 3}$ (MeV) | $g_{\sigma 4}$ | $g_{\omega 4}$ | $g_{\rho 4}$ |
|---|---|---|---|---|---|---|---|
| NHA$^{2.50}$ | 9.684 | 10.597 | 5.300 | 150.0 | 380.0 | 380.0 | 4.00 |
| NHA$^{2.00}$ | 8.613 | 7.933 | 5.990 | 500.0 | 2500.0 | 2500.0 | 4.00 |

| $g_{\sigma\omega}$ | $g_{\sigma\rho}$ | $g_{\omega\rho}$ | $g_{\sigma\sigma N}$ | $g_{\sigma\omega N}$ | $g_{\sigma\rho N}$ | $g_\sigma^*$ | $g_\omega^*$ | $g_\rho^*$ |
|---|---|---|---|---|---|---|---|---|
| 210.0 | −62.0 | −62.0 | −0.045 | 0.012 | 0.043 | 9.056 | 10.932 | 7.702 |
| 1400.0 | −30.0 | −30.0 | −0.070 | 0.050 | 0.059 | 7.933 | 8.904 | 8.282 |

| | $M_N^*/M$ | $m_\sigma^*/m_\sigma$ | $m_\omega^*/m_\omega$ | $m_\rho^*/m_\rho$ | $K$ | $a_4$ | $M_{max}$ | $\mathcal{E}_c$ | $I$ | R |
|---|---|---|---|---|---|---|---|---|---|---|
| NHA$^{2.50}$ | 0.73 | 1.07 | 1.08 | 1.00 | 218 | 30.0 | 2.50 | 1.78 | 371 | 12.5 |
| NHA$^{2.00}$ | 0.84 | 1.18 | 1.23 | 1.00 | 161 | 30.0 | 2.00 | 2.11 | 192 | 11.6 |

The results of the linear $\sigma$-$\omega$ Hartree approximation (LHA)[24], and the current nonlinear $\sigma$-$\omega$-$\rho$ Hartree approximations, NHA$^{2.50}$ and NHA$^{2.00}$ for $M_{max} = 2.50\ M_\odot$ and 2.00 $M_\odot$, are compared. $M_N^*$ is the effective mass of nucleon and note that the difference of $M_n^*$ and $M_p^*$ is less than 0.1 % in all densities; $m_\sigma^*$, $m_\omega^*$ and $m_\rho^*$ are effective masses of $\sigma$, $\omega$ and $\rho$ mesons, respectively. The masses are fixed as $M = 939$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV and $m_\rho = 770$ MeV[24]. $M_{max}$ is the maximum mass in the solar mass unit ($M_\odot$) and $\mathcal{E}_C$ ($10^{15}$ g/cm$^3$) is the central energy density; $I$ is the inertial mass ($M_\odot \cdot km^2$) and $R$ ($km$) is the radius of a neutron star. The coupling constants are adjusted to reproduce the values: $\mathcal{E}/\rho_B - M = -15.75$ MeV, at $k_F = 1.30$ fm$^{-1}$, and $a_4 = 30.0$ MeV. The equations of state are connected to the one of pure neutron matter, and the maximum masses of pure-neutron stars are fixed as, $M_{max}(n) = 2.50\ M_\odot$ and $M_{max}(n) = 2.00\ M_\odot$ [24]–[28]. The compressibilities, $K$, for NHA$^{2.50}$ and NHA$^{2.00}$ are numerically evaluated as the lower bound of compressibility required to support the maximum masses of neutron stars, respectively[23].

correlations between symmetric and asymmetric nuclear matter.

The energy density, $\mathcal{E}_{NHA}$, and pressure, $p_{NHA}$, can be derived by applying the baryon Green's function $G_D(k)$ and the free nonlinear meson propagators with effective masses of mesons, $D_\sigma^{(0)}(k)$, $D_\omega^{(0)\ \mu\nu}(k)$ and $D_{\rho\ ab}^{(0)\ \mu\nu}(k)$, to tadpole diagrams[24]. The following results will be obtained:

$$\begin{aligned}\mathcal{E}_{NHA} = & \frac{\zeta}{(2\pi)^3}\int^{k_F} d^3k E(k) + \frac{m_\sigma^2}{2}\phi_0^2 + \frac{g_{\sigma 3}}{3!}\phi_0^3 + \frac{g_{\sigma 4}}{4!}\phi_0^4 \\ & - \frac{m_\omega^2}{2}V_0^2 - \frac{g_{\omega 4}}{4!}V_0^4 - \frac{g_{\sigma\omega}}{4}\phi_0^2 V_0^2 - \Big(\frac{m_\rho^2}{2} + \frac{g_{\rho 4}}{4!}R_0^2 + \frac{g_{\sigma\rho}}{4}\phi_0^2 + \frac{g_{\omega\rho}}{4}V_0^2\Big)R_0^2 \ ,\end{aligned} \tag{3.1}$$

$$\begin{aligned}p_{NHA} = & \frac{1}{3}\frac{\zeta}{(2\pi)^3}\int^{k_F} d^3k \frac{k^2}{E^*(k)} - \frac{m_\sigma^2}{2}\phi_0^2 - \frac{g_{\sigma 3}}{3!}\phi_0^3 - \frac{g_{\sigma 4}}{4!}\phi_0^4 \\ & + \frac{m_\omega^2}{2}V_0^2 + \frac{g_{\omega 4}}{4!}V_0^4 + \frac{g_{\sigma\omega}}{4}\phi_0^2 V_0^2 + \Big(\frac{m_\rho^2}{2} + \frac{g_{\rho 4}}{4!}R_0^2 + \frac{g_{\sigma\rho}}{4}\phi_0^2 + \frac{g_{\omega\rho}}{4}V_0^2\Big)R_0^2 \ ,\end{aligned} \tag{3.2}$$

and note that the mean-fields of mesons are given by, $\phi_0 = (M - M^*(k_F))/g_\sigma^*$, $V_0 = (g_\omega^*/m_\omega^{*2})\rho_B$ and $R_0 = (g_\rho^*/2m_\rho^{*2})\rho_3$. The effective masses and coupling constants, (2.7) and (2.12), are not manifest in the eqs. (3.1) and (3.2); however, they are essential in order to prove self-consistency of equations of motion, energy density and self-energies.

The energy density, pressure and single particle energy, ($\mathcal{E}_{NHA}$, $p_{NHA}$, $E(k)$), satisfy the thermodynamic relation:

$$\mathcal{E}_{NHA} + p_{NHA} = \rho_B E(k_F) \ . \tag{3.3}$$

In order to obtain (3.3), the single particle energy, $E(k) = E^*(k) - \Sigma^0(k_F)$ where $\Sigma^0(k_F) = -g_\omega^* V_0$, is used to partially integrate the first term of (3.1). By adding the result to the first term of (3.2), one obtains the right-hand side of (3.3); the rest of the terms cancel one another. Note that in isospin-asymmetric matter, one should have $\rho_B E(k_F) = \rho_p E_p(k_{F_p}) + \rho_n E_n(k_{F_n})$, where $E_{\binom{p}{n}} = E^*_{\binom{p}{n}} + g_\omega^* V_0 \pm (g_\rho^*/2)R_0$, and $E_{\binom{p}{n}}$ denotes $E_p(k_{F_p})$, $E_n(k_{F_n})$, respectively. The equality of chemical potential and the single particle energy, $\mu = E(k_F)$, is also proved by differentiating the energy density, $\mathcal{E}_{NHA}$, with respect to the baryon density, $\rho_B$, and using equations of motion for mesons (2.8), which results in,

$$\mu = \frac{\partial \mathcal{E}_{NHA}}{\partial \rho_B} = E(k_F) \ . \tag{3.4}$$

The exact result (3.4) proves the Hugenholtz-Van Hove theorem to an approximation. It is essential to calculate Fermi-liquid properties and Landau parameters of nuclear matter[16] as well as for the definition of the density of nuclear matter saturation, where the pressure vanishes: $p_{NHA} = 0$.

The functional derivative of energy density, $\mathcal{E}_{NHA}(\phi_0, V_0, R_0, n_i)$, with respect to the baryon number distribution, $n_i$, is given by:

$$\frac{\delta\mathcal{E}_{NHA}}{\delta n_i} = E(k_i) + \sum_i \Big(\frac{\delta\mathcal{E}_{NHA}}{\delta\phi_0}\frac{\delta\phi_0}{\delta n_i} + \frac{\delta\mathcal{E}_{NHA}}{\delta V_0}\frac{\delta V_0}{\delta n_i} + \frac{\delta\mathcal{E}_{NHA}}{\delta R_0}\frac{\delta R_0}{\delta n_i}\Big)\ . \tag{3.5}$$

The self-consistent condition of the theory of conserving approximations requires: $\dfrac{\delta\mathcal{E}_{NHA}}{\delta\phi_0} = 0$, $\dfrac{\delta\mathcal{E}_{NHA}}{\delta V_0} = 0$ and $\dfrac{\delta\mathcal{E}_{NHA}}{\delta R_0} = 0$ [23]. The condition independently generates meson equations of motion (2.8) and determines self-energies of the approximation. One can examine that the self-energies calculated by propagators, (2.9) $\sim$ (2.11), and by the condition of conserving approximations become equivalent, only if the effective masses and effective coupling constants of mesons are given by (2.7) and (2.12). The nonlinear mean-field approximation becomes thermodynamically consistent, relativistic, field-theoretical approximation, with the effective masses of mesons and coupling constants. The self-consistency with effective masses and coupling constants, ($M_N^*$, $m_\sigma^*$, $m_\omega^*$, $m_\rho^*$) and ($g_\sigma^*$, $g_\omega^*$, $g_\rho^*$), is finally proved by the condition of conserving approximations[18, 23, 38]. The results will also signify that nonlinear mean-field approximations naturally generate effective masses and coupling constants for self-consistency. The current functional differential approach can be applied to Hartree-Fock, Ring and other effective approximations and models. The resulting self-consistent approximations are essential to analyze density-dependent correlations between properties of nuclear matter and neutron stars, signals of phase transition between hadron and quark matter[11].

# 4 Properties of nuclear matter and neutron stars

The density-dependent correlations between properties of nuclear matter and neutron stars have been pointed out[32]–[37]; this is also confirmed in the nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximations[23, 38]. We will apply the results on pure-neutron stars, $M_{star}(n)$, and isospin-asymmetric neutron stars, $M_{star}(p, n, e)$ composed of protons, neutrons and electrons. The binding energies by the linear $\sigma$-$\omega$ Hartree approximation (LHA)[24], and the current nonlinear Hartree approximations, denoted as NHA$^{2.50}$ and NHA$^{2.00}$ which generate the maximum mass of neutron stars: $M_{max} = 2.50\ M_\odot$ and $2.00\ M_\odot$, are compared in the Fig. 4. They are numerically evaluated by solving equations of motion iteratively with a given density $\rho_B$; then, the single particle energy, energy density and pressure are calculated. The properties of neutron stars are evaluated by employing the equation of state for neutron matter. The coupling constants are adjusted in order to produce saturation properties, $\mathcal{E}/\rho_B - M = -15.75$ MeV,

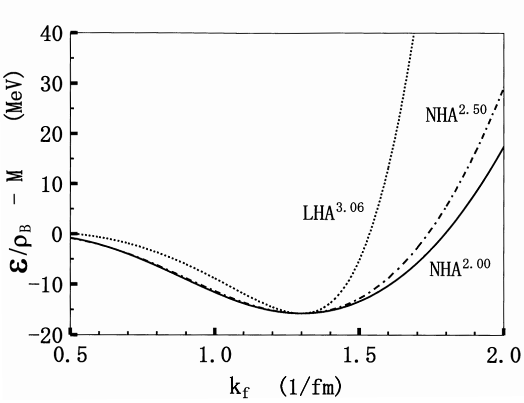


Fig. 4. The binding energy curves of $(n, p)$ symmetric nuclear matter. The linear $\sigma$-$\omega$ Hartree approximation (LHA), and the nonlinear Hartree approximations, $\mathrm{NHA}^{2.50}$ and $\mathrm{NHA}^{2.00}$ which produce the maximum masses of neutron stars: $M_{max} = 2.50\ M_{\odot}$ and $M_{max} = 2.00\ M_{\odot}$, are shown respectively (see, the Table 1).

at $k_F = 1.30\ \mathrm{fm}^{-1}$, the symmetry energy $a_4 = 30.0$ MeV. Simultaneously, the minimum value of compressibility, $K$, is investigated to the maximum mass, $M_{max} = 2.50\ M_{\odot}$ and $2.00\ M_{\odot}$, of neutron stars[27, 28]. The equation of state and properties of nuclear matter compatible with the mass of hadronic neutron stars will be explicitly constrained through the density-dependent effective masses and coupling constants.

The compressibility is calculated by

$$K = 9\rho_B \frac{\partial^2 \mathcal{E}}{\partial \rho_B^2} = 9\rho_B \left( \frac{\partial \mu}{\partial \rho_B} \right) , \tag{4.1}$$

where $\mu$ is the chemical potential and equal to the Fermi energy, $\mu = E(k_F)$, since HV theorem and Landau's hypothesis for quasiparticles are maintained exactly in the current mean-field approximation. Coupling constants and Fermi-liquid properties of nuclear matter and neutron stars are listed in the Table 1. The linear Hartree approximation (LHA) gives $K = 530$ MeV and $M_{max} = 3.07\ M_{\odot}$. The nonlinear Hartree approximation, $\mathrm{NHA}^{2.50}$, shows that the lower bound of compressibility is about $K \sim 220$ MeV, whereas $\mathrm{NHA}^{2.00}$ exhibits that the lower bound of compressibility is about $K \sim 160$ MeV. The larger the compressibilities, the steeper the binding energy curves at saturation will correspond: $K(\mathrm{LHA}^{3.06}) > K(\mathrm{NHA}^{2.50}) > K(\mathrm{NHA}^{2.00})$. The results indicate that the density-dependence and retardation effects will soften the equation of state in the mean-field approximation of hadrons.

The coupling constants which either increase or decrease the values of both $K$ and $a_4$ will have the same tendency to the value of $M_{max}$. The $\sigma$-$\omega$ mixed-interaction term, $g_{\sigma\omega}$, is an exception that decreases the value of K but increases the values of both $a_4$ and $M_{max}$, which indicates the $\sigma$-$\omega$ mixed interaction may simulate distinct many-body interactions at normal density. The softer equation of state will decrease the value of compressibility, $K$, and simultaneously, decrease the maximum mass of neutron stars, $M_{max}$ and symmetry energy, $a_4$. Nonlinear interactions will connect physical quantities closely to one another that sensitive adjustments are needed to reproduce experimentally desired values of $K$, $a_4$ and $M_{max}$. The properties of nuclear matter and neutron stars are modified mainly by changing $g_{\sigma 4}$, $g_{\omega 4}$, the mixed-interaction term $g_{\sigma\omega}$ and effective coupling constants, $g^*_\sigma$ and $g^*_\omega$. The coupling constant, $g_{\sigma\rho N}$, contributes mainly to $a_4$ and $M_{max}$. The effective coupling constants, $g^*_\sigma$ and $g^*_\omega$, change slowly with density, but they contribute significantly to properties of nuclear matter at saturation, whereas properties of neutron stars are mainly determined by effective masses of hadrons rather than effective coupling constants.

The symmetry energy is calculated by

$$a_4 = \frac{1}{2}\rho_B \left[ \left[ \frac{\partial^2 \mathcal{E}}{\partial \rho_3^2} \right]_{\rho_B} \right]_{\rho_3=0} , \tag{4.2}$$

where $\rho_3$ is the difference between the proton and neutron density: $\rho_3 = \rho_p - \rho_n = (k^3_{F_p} - k^3_{F_n})/3\pi^2$ at a fixed baryon density, $\rho_B = \rho_p + \rho_n = 2k^3_F/3\pi^2$. The $\rho$-meson mean-field $R_0$ and self-energies are calculated by,

$$\frac{g^*_\rho}{2} R_0 = -\Sigma^0_{\rho p} = \Sigma^0_{\rho n} = \frac{g^{*2}_\rho}{4m^{*2}_\rho}\rho_3 , \tag{4.3}$$

where $g^*_\rho = g_\rho + 2g_{\sigma\rho N}\phi_0$. In symmetric nuclear matter ($\rho_3 = 0$), the $\rho$-meson mean-field, $R_0$, $\sigma$-$\rho$ and $\omega$-$\rho$ interactions vanish, and $m^*_\rho = m_\rho$ will be restored. The effective mass of $\rho$-meson is similar to the bare mass: $m^*_\rho \sim m_\rho$ at saturation density so that contributions to the effective mass of $\rho$-meson from nonlinear interactions are negligible at saturation density in the level of mean-field approximation. However, nonlinear interactions will cause to decrease $m^*_\rho$ suddenly in high densities, which induces the saturation of symmetry energy in isospin-asymmetric nuclear matter. The density-dependent effective mass of $\rho$-meson, $m^*_\rho$, is an important factor in high densities.

Coupling constants should be carefully adjusted so that one can obtain solutions in the density range, $0 < k_F \leq 3.0\,\mathrm{fm}^{-1}$, where the equation of state is important to determine the mass of neutron stars. The conditions of conserving approximation and consequently, self-consistent condition, $\partial\mathcal{E}/\partial n_i = E(k_i)$, are essential to evaluate eqs. (4.1) $\sim$ (4.2) of compressibility and symmetry energy. The masses, moment of inertia of neutron stars are calculated by Tollman-Oppenheimer-Volkoff (TOV) equation[39]–[41]. The data of NHA$^{2.00}$ indicate that the compressibility, $K > 160$ MeV, is necessary in order to reproduce all the observed masses of hadronic neutron stars; $K \sim 160$ MeV is the lower bound to support all the hadronic neutron stars in the nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation. The lower bound of compressibility changes about 26% ($K = 218 \rightarrow 161$ MeV), as the maximum mass of neutron stars is changed 20% ($M_{max} = 2.50 \rightarrow 2.00\ M_{\odot}$); this may exhibit the strong correlation of compressibility to the maximum mass of neutron stars. All the results are consistent with the analysis of acceptable configuration of the central energy and maximum masses of neutron stars[28]. The approximations LHA, NHA$^{2.50}$, and NHA$^{2.00}$ produce distinct results for properties of nuclear and neutron stars, which may be distinguished in the course of the accurate examinations of nuclear experimental data and accumulating data of neutron stars.

# 5 The beta-equilibrium $(n, p, e)$ nuclear matter in the nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation

The current nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation will be applied to the isospin asymmetric, $\beta$-equilibrium nuclear matter. Then, the density-dependent correlations between isospin symmetric and asymmetric nuclear matter will be examined. The isospin-asymmetry is induced by the electric field; the lepton field is not directly coupled with hadronic fields, but it is important to determine Fermi-liquid properties of nuclear matter, equation of state and properties of neutron stars. The values of all coupling constants are to be adjusted to reproduce saturation properties for $(n, p)$-symmetric nuclear matter, $\mathcal{E}/\rho_B - M = -15.75$ MeV, at $k_F = 1.30$ fm$^{-1}$, symmetry energy, $a_4 = 30.0$ MeV[23], and simultaneously, the equations of state have to produce the maximum masses of isospin-asymmetric neutron stars; the maximum masses are supposed to be: $M_{max}(n, p, e) = 2.50\ M_{\odot}$ and $M_{max}(n, p, e) = 2.00$; the coupling constants are also adjusted to produce the minimum value of compressibility, $K$.

The nonlinear mean-field lagrangian with effective mass and vertex interactions is denoted by $\mathcal{L}'_{NHA}$ and applied to $(n, p, e)$ nuclear matter with the electron field as,

$$\begin{aligned}\mathcal{L}'_{NHA} =& \mathcal{L}_{NHA} + \bar{\psi}_N \Big[ -g_{\sigma\omega N}\phi_0\gamma_0 V_0 - g_{\sigma\rho N}\phi_0\gamma_0\tau_3 R_0 + \frac{g_{\sigma\sigma N}}{2}\phi_0^2) \Big] \psi_N \\ &+ \bar{\psi}_e (i\gamma_\mu \partial^\mu - m_e)\psi_e \ .\end{aligned} \tag{5.1}$$

The energy density and pressure of isospin and charge asymmetric nuclear matter are calculated by way of the energy-momentum tensor, (see also [26]) as:

$$
\begin{aligned}
\mathcal{E}_{NHA} = & \sum_{B=n,p} \frac{1}{\pi^2} \int_0^{k_{F_B}} dk k^2 E_B(k) + \frac{m_\sigma^2}{2}\phi_0^2 + \frac{g_{\sigma 3}}{3!}\phi_0^3 + \frac{g_{\sigma 4}}{4!}\phi_0^4 \\
& - \frac{m_\omega^2}{2} V_0^2 - \frac{g_{\omega 4}}{4!} V_0^4 - \frac{g_{\sigma\omega}}{4}\phi_0^2 V_0^2 \\
& - \Big(\frac{m_\rho^2}{2} + \frac{g_{\rho 4}}{4!} R_0^2 + \frac{g_{\sigma\rho}}{4}\phi_0^2 + \frac{g_{\omega\rho}}{4} V_0^2\Big) R_0^2 + \frac{1}{\pi^2}\int_0^{k_{F_e}} dk k^2 E_e^*(k) \ ,
\end{aligned} \tag{5.2}
$$

$$
\begin{aligned}
p_{NHA} = & \frac{1}{3\pi^2} \sum_{B=n,p} \int_0^{k_{F_B}} dk \frac{k^4}{E_B^*(k)} - \frac{m_\sigma^2}{2}\phi_0^2 - \frac{g_{\sigma 3}}{3!}\phi_0^3 - \frac{g_{\sigma 4}}{4!}\phi_0^4 \\
& + \frac{m_\omega^2}{2} V_0^2 + \frac{g_{\omega 4}}{4!} V_0^4 + \frac{g_{\sigma\omega}}{4}\phi_0^2 V_0^2 \\
& + \Big(\frac{m_\rho^2}{2} + \frac{g_{\rho 4}}{4!} R_0^2 + \frac{g_{\sigma\rho}}{4}\phi_0^2 + \frac{g_{\omega\rho}}{4} V_0^2\Big) R_0^2 + \frac{1}{3\pi^2}\int_0^{k_{F_e}} dk \frac{k^4}{E_e^*(k)} \ ,
\end{aligned} \tag{5.3}
$$

where $k_{F_B}$ denotes the Fermi-momentum for protons and neutrons, $k_{F_p}$ and $k_{F_n}$, and $E_e(k) = (\mathbf{k}^2 + m_e^2)^{1/2}$. One can check that for a given baryon density, $\rho_B = \rho_p + \rho_n$, and charge-neutrality, $\rho_p = \rho_e$, the thermodynamic relation

$$
\mathcal{E}_{NHA} + p_{NHA} = \rho_B E_n(k_{F_n}) = \rho_n \mu_n + \rho_p \mu_p + \rho_e \mu_e \ , \tag{5.4}
$$

with $\mu_n = \mu_p + \mu_e$, and the chemical potential, $\mu_n = \partial \mathcal{E}_{NHA}/\partial \rho_B = E_n(k_{F_n})$, are exactly satisfied. The self-consistent effective coupling constants ($g_\sigma^*$, $g_\omega^*$, $g_\rho^*$) and effective masses of mesons ($m_\sigma^*$, $m_\omega^*$, $m_\rho^*$) are given by (2.7) and (2.12) in isospin-asymmetric $(n,p,e)$ matter. The functional derivatives of $\mathcal{E}_{NHA}$ with respect to self-energies, or fields, can be directly performed, and thermodynamic consistency of the current approximation is proved in all densities.

Charge neutrality and asymmetry are generally considered by introducing the ratio, $r_{ep}$, of electron to proton density, defined by $\rho_e = r_{ep}\rho_p$, which yields the phase-equilibrium condition derived by the method of Lagrange multiplier,

$$
\mu_n = \mu_p + r_{ep}\mu_e \ , \tag{5.5}
$$

where charge neutrality is expressed as $r_{ep} = 1$, and when the electron-proton ratio is $0 < r_{ep} < 1$, the system is in charge-asymmetric, neutron-rich nuclear matter; $r_{ep} = 0$ expresses the isospin symmetric ($n$, $p$) matter. The binding energies of isospin-symmetric nuclear matter, $\mathrm{NHA}^{2.50}(n,p)$ and $\mathrm{NHA}^{2.00}(n,p)$, and charge-neutral, isospin-asymmetric nuclear matter, $\mathrm{NHA}^{2.50}(n,p,e)$ and $\mathrm{NHA}^{2.00}(n,p,e)$, are shown in the Fig. 5, and properties of symmetric nuclear matter and isospin-asymmetric neutron stars are listed in the Table 2. The isospin-asymmetric, charge-neutral binding energies of $(n,p,e)$ matter are not

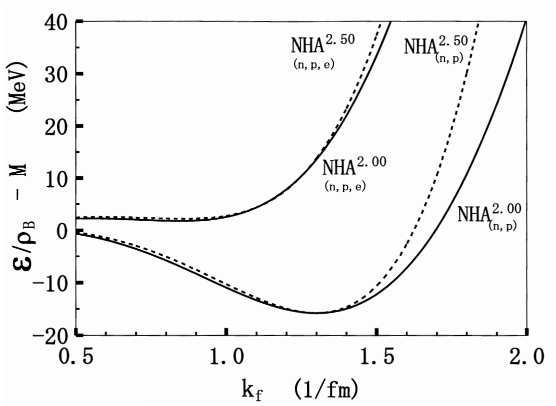


Fig. 5. The binding energies of $(n, p)$-symmetric and $(n, p, e)$-asymmetric nuclear matter. The repulsive contributions generated by electrons and isospin-asymmetry render the binding energy of $(n, p, e)$-asymmetric nuclear matter repulsive in all densities.

bounded as seen in the Fig. 5 and give hard equations of state compared to the isospin-symmetric nuclear matter NHA$(n, p)$.

The binding energies in Fig. 5 indicate that the electric field and isospin-asymmetry produce repulsive forces which uphold binding energies of $(n, p)$

Table 2.
Properties of isospin-symmetric nuclear matter and isospin-asymmetric neutron stars, $M_{star}(n, p, e)$, in beta-equilibrium.

| | $g_\sigma$ | $g_\omega$ | $g_\rho$ | $g_{\sigma 3}$ (MeV) | $g_{\sigma 4}$ | $g_{\omega 4}$ | $g_{\rho 4}$ |
|---|---|---|---|---|---|---|---|
| $\mathrm{NHA}^{2.50}$ | 9.326 | 10.421 | 4.765 | 10.0 | 20.0 | 20.0 | 4.00 |
| $\mathrm{NHA}^{2.00}$ | 7.629 | 6.675 | 5.810 | 20.0 | 80.0 | 80.0 | 4.00 |

| $g_{\sigma\omega}$ | $g_{\sigma\rho}$ | $g_{\omega\rho}$ | $g_{\sigma\sigma N}$ | $g_{\sigma\omega N}$ | $g_{\sigma\rho N}$ | $g^*_\sigma$ | $g^*_\omega$ | $g^*_\rho$ |
|---|---|---|---|---|---|---|---|---|
| 18.0 | −18.0 | −18.0 | −0.018 | 0.013 | 0.048 | 9.063 | 10.800 | 7.567 |
| 72.0 | −42.0 | −42.0 | −0.070 | 0.020 | 0.057 | 6.879 | 7.103 | 8.252 |

| | $M^*_N/M$ | $m^*_\sigma/m_\sigma$ | $m^*_\omega/m_\omega$ | $K$ | $a_4$ | $M_{max}$ | $\mathcal{E}_c$ | $I$ | R |
|---|---|---|---|---|---|---|---|---|---|
| $\mathrm{NHA}^{2.50}$ | 0.70 | 1.02 | 1.01 | 329 | 30.0 | 2.50 | 1.60 | 369 | 12.8 |
| $\mathrm{NHA}^{2.00}$ | 0.84 | 1.06 | 1.02 | 256 | 30.0 | 2.00 | 1.98 | 193 | 11.7 |

Since the equation of state of isospin-asymmetric, beta-equilibrium nuclear matter becomes softer than that of pure-neutron matter, a harder equation of state is required[38] in order to keep the maximum masses, $M_{max}(n, p, e) = 2.50\ M_\odot$ and $M_{max}(n, p, e) = 2.00\ M_\odot$. This is confirmed by larger values of compressibilities, $K = 329$ and 256 MeV, compared to the Table 1.

symmetric nuclear matter. Since the saturation of binding energy corresponds to the existence of self-binding, stable nuclear matter or nuclei, the isospin-asymmetric $(n, p, e)$ matter which is energetically higher than $(n, p)$ symmetric matter would be expected to convert to the $(n, p)$ symmetric matter by way of the reverse beta-reaction as, $p + e^- \to n + \nu$. The conversions from isospin-asymmetric $(n, p, e)$ matter to stable, self-bound nuclear matter or nuclei could be possible on the surface of neutron stars where the low-density nuclear matter and strong gravitational field could support the reverse beta-reaction; the surface of neutron stars is the place for nucleation. Charge-neutrality may be a strict constraint to examine saturation properties of isospin-asymmetric nuclear matter; in fact, when the electron-proton ratio is set as, $0 < r_{ep} < 1$, the saturation of binding energy is obtained. Based on the analyses, we will derive binding energy and density, compressibility and symmetry energy at saturation of isospin-asymmetric nuclear matter and investigate density-dependent correlations between saturation properties of isospin symmetric and asymmetric nuclear matter.

# 6 The saturation properties of isospin asymmetric, neutron-rich nuclear matter $\mathcal{E}(n, p, e)$

The neutron-rich nuclei and isospin-asymmetric nuclei are abundant in astronomical objects, such as the solar system, stars, cosmic rays which are important matter to examine thermonuclear evolution in stars and galaxies[42]. We will investigate density-dependent correlations between symmetric nuclear matter and isospin-asymmetric, neutron-rich nuclear matter in beta-equilibrium $(n, p, e)$. The isospin asymmetric, beta-equilibrium $(n, p, e)$ nuclear matter provides important constraints on saturation properties of symmetric nuclear matter, such as compressibility and symmetry energy and properties of neutron stars. In the current analysis, the isospin-asymmetric, charge-neutral nuclear matter is defined with the eq. (5.5) ($r_{ep} = 1$) of phase-equilibrium condition. For example, the isospin-asymmetric nuclear matter may be the stable nuclear matter composed of magic nuclei, such as Fe $(Z, A, N = 25, 52, 27)$, Sr $(Z, A, N = 38, 88, 50)$ and Ba $(Z, A, N = 56, 138, 82)$ and Pb $(Z, A, N = 82, 208, 126)$; the correlations and saturation properties, binding energies and densities will be computed and examined.

The isospin-asymmetry generated by the presence of electric field produces repulsive contributions to energy density, which leads to unbound energy density as shown in NHA$(n, p, e)$ of the Fig. 5. The contribution of the electric field is important for saturation density, but it is negligible for high-density regions. The equation of state in beta-equilibrium, NHA$(n, p, e)$, becomes harder than that of the isospin-symmetric state, NHA$(n, p)$. This is examined

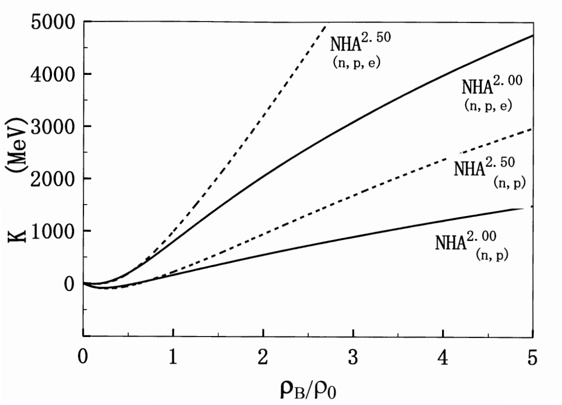


Fig. 6. Compressibilities of $(n, p, e)$-asymmetric and $(n, p)$ symmetric nuclear matter are compared. The equations of state of $(n, p, e)$-asymmetric matter are harder than those of $(n, p)$ symmetric matter.

from the Fig. 6, which exhibits compressibilities with respect to baryon densities; the harder equation of state corresponds to a larger compressibility. The electric field and $\rho$-meson interactions generate similar positive contributions to symmetry energy, $a_4$, for saturation density, but the $\rho$-meson produces strongly negative contributions, which is examined from the abrupt change of effective mass of $\rho$-meson and the decrease of symmetry energy, $a_4$, for high-density regions. The symmetry energy will increase monotonically in the

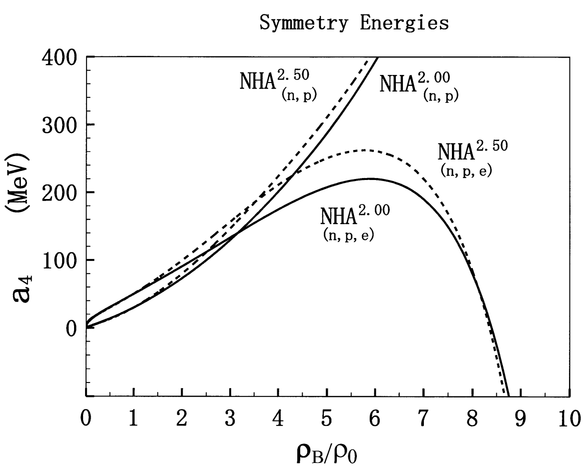


Fig. 7. Symmetry energies in isospin-symmetric and isospin-asymmetric nuclear matter. Note that symmetry energy in isospin-symmetric $(n, p)$ matter with nonlinear interactions is monotonically increasing in all densities, whereas it saturates in isospin-asymmetric $(n, p, e)$ matter.

isospin-symmetric nuclear matter, whereas it will saturate and decrease in the isospin-asymmetric nuclear matter, which is shown in the Fig. 7. The saturation of symmetry energy for high-density regions comes from the contributions of both the nonlinear interactions of $\rho$-meson sector and isospin asymmetry[38].

The beta-equilibrium nuclear matter is defined with effective coupling constants and masses which are adjusted and fixed to reproduce saturation properties in isospin-symmetric nuclear matter, $\mathcal{E}/\rho_B - M = -15.75$ MeV, at $k_F = 1.30$ fm$^{-1}$, and maximum masses of isospin-asymmetric neutron stars, $M_{max}(n,p,e) = 2.50$ and $2.00\ M_\odot$ with the equation of state in NHA$(n,p,e)$. As it is explained briefly in the sec. 5, the binding energy of isospin-asymmetric nuclear matter with charge neutrality would be expected to convert into stable isospin-symmetric nuclear matter by radiating neutrino through the beta-equilibrium process ($n + \nu \rightleftharpoons p + e$). The beta-equilibrium neutrino-radiation by producing stable neutron-rich nuclei may occur in the low density regions, such as on the surface or in the atmosphere of a neutron star; hence, the isospin-asymmetric neutron star is like a giant furnace which produces heavy nuclei on the surface. If neutron stars are constant source of neutrino-radiation after the supernova explosion and a long-time scale neutrino-cooling process is experimentally observed, the beta-equilibrium neutrino-radiation may be considered. The quantitative data of the beta-equilibrium neutrino-radiation would provide significant information on the structure of the neutron stars and theoretical models of nuclear physics.

Let us consider neutron-rich matter ideally composed of only with a specific magic nuclei. The isospin-asymmetric, neutron-rich nuclear matter corresponding to magic number nuclei is calculated and compared with isospin symmetric nuclear matter. For examples, nuclear matter composed of magic nuclei, such as Fe $(Z,A,N = 25,52,27)$, Sr $(Z,A,N = 38,88,50)$, Ba $(Z,A,N = 56,138,82)$ and Pb $(Z,A,N = 82,208,126)$ are considered. The ratios of proton numbers to neutron, $r_p = Z/A$, will be used to calculate $\rho_p$ and $\rho_n$ of isospin-asymmetric matter, and they are: $r_p = 0.481$ (Fe), $r_p = 0.432$ (Sr), $r_p = 0.406$ (Ba) and $r_p = 0.394$ (Pb), respectively. For a given baryon density $\rho_B$, the proton, neutron and electron density, $\rho_p$, $\rho_n$ and $\rho_e$, must be determined self-consistently with the condition of charge-neutrality and $M_{max}(n,p,e) = 2.50$ and $2.00\ M_\odot$; the binding energies and densities at saturation of neutron-rich nuclear matter composed with magic nuclei and density-dependent effective masses of hadrons, compressibility and symmetry energy, are computed. The results are listed in the Table 3, and binding energies of Sr and Pb are shown in the Fig. 8a and 8b.

The neutron-rich nuclear matter of magic nuclei gives slightly smaller binding energy and density compared to those of isospin-symmetric nuclear matter, and the saturation densities of isospin-asymmetric matter exist only in low densities. The current results also suggest that neutron-rich heavy nuclei as well as

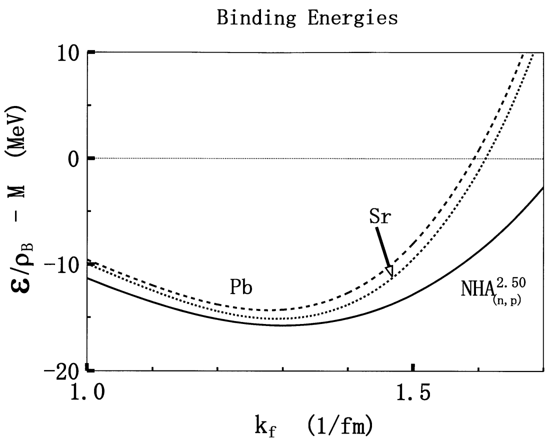


Fig. 8. (a) The binding energies of neutron-rich nuclear matter corresponding to Pb and Sr are shown. The equations of state produce the maximum mass of neutron stars, $M_{max} = 2.50\ M_\odot$ (see, Table 3).

exotic nuclei might abundantly exist on the surface of neutron stars. The neutron stars are such a giant laboratory which is difficult to set up on the earth; therefore, they are very interesting objects for the theory and experiment in nuclear, high-energy and astrophysics. The current nonlinear mean-field ap-

Table 3
Properties of isospin-asymmetric nuclear matter of magic nuclei.

| | $k_F$ | $\mathcal{E}/\rho_B - M_N$ | $M_N^*/M$ | $m_\sigma^*/m_\sigma$ | $m_\omega^*/m_\omega$ | $m_\rho^*/m_\rho$ | $K$ | $a_4$ |
|---|---|---|---|---|---|---|---|---|
| NHA$^{2.50}$ | 1.30 | −15.75 | 0.70 | 1.02 | 1.01 | 1.00 | 329 | 30.0 |
| Fe | 1.30 | −15.68 | 0.70 | 1.02 | 1.01 | 0.99 | 329 | 29.9 |
| Sr | 1.29 | −15.12 | 0.71 | 1.02 | 1.01 | 0.99 | 322 | 29.2 |
| Ba | 1.28 | −14.60 | 0.71 | 1.02 | 1.01 | 0.99 | 315 | 28.5 |
| Pb | 1.28 | −14.31 | 0.72 | 1.02 | 1.01 | 0.99 | 311 | 28.1 |
| NHA$^{2.00}$ | 1.30 | −15.75 | 0.84 | 1.06 | 1.02 | 1.00 | 256 | 30.0 |
| Fe | 1.30 | −15.69 | 0.84 | 1.06 | 1.02 | 0.99 | 256 | 29.7 |
| Sr | 1.29 | −15.12 | 0.85 | 1.06 | 1.02 | 0.99 | 251 | 29.0 |
| Ba | 1.28 | −14.61 | 0.85 | 1.06 | 1.02 | 0.99 | 246 | 28.3 |
| Pb | 1.28 | −14.32 | 0.85 | 1.06 | 1.02 | 0.99 | 243 | 27.9 |

Properties of isospin-asymmetric nuclear matter corresponding to infinite matter of magic nuclei are listed. The nonlinear coupling constants determined in the Table 2 for the equation of state, $M_{max}(n,p,e) = 2.50\ M_\odot$ and $M_{max}(n,p,e) = 2.00\ M_\odot$, are employed and listed for comparison. The binding energies, $\mathcal{E}/\rho_B - M_N$ (MeV), and Fermi-momentum, $k_F$ (fm$^{-1}$), the effective masses of hadrons, compressibility and symmetry energy are compared.

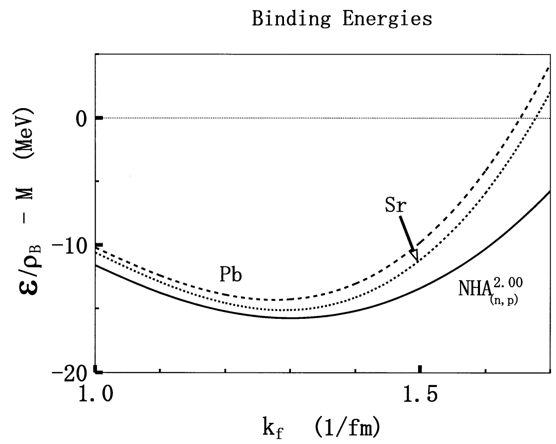


Fig. 8. (b) The binding energies of neutron-rich nuclear matter corresponding to Pb and Sr are shown. The equations of state produce the maximum mass of neutron stars, $M_{max} = 2.00\ M_\odot$ (see, Table 3).

proximation explicitly exhibits the strong density-dependent correlations between saturation properties of isospin-asymmetric, neutron-rich nuclear matter and those of symmetric nuclear matter. The *binding energy of neutron-rich nuclear matter at saturation* would be important constraints for both properties of $(n, p)$ symmetric nuclear matter and $(n, p, e)$-, $(n, p, \text{hyperon}, e)$-neutron stars. It is desired to investigate saturation properties of neutron-rich nuclear matter, or isospin-asymmetric nuclear matter in general, experimentally and theoretically, in order to understand models of nuclear physics.

# 7 Conclusions and Remarks

We have employed a nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation[23] in order to simulate complicated many-body interactions of hadrons and studied Fermi-liquid properties of nuclear matter and neutron stars. Nonlinear interactions are renormalized self-consistently with the conditions of thermodynamic consistency. The requirement of thermodynamic consistency results in the introduction of effective masses and effective coupling constants of nucleons and mesons. The contributions of nonlinear interactions to physical quantities (effective masses, $M^*$, $m^*_\sigma$, $m^*_\omega$, $m^*_\rho$, compressibility, $K$ and symmetry energy, $a_4$) and neutron stars (maximum mass $M_{max}$, central energy density $\mathcal{E}_c$, moment of inertia $I$, and radius $R$) are discussed in isospin symmetric $(n, p)$ and asymmetric $(n, p, e)$ matter, in order to examine the effect of isospin-variation, density and nonlinear interactions to physical quantities. The nonlinear mean-field models seem to simulate properties of nuclear and neutron matter reasonably well[25]. However, since the effective masses of mesons and nucleons are self-consistently confined in order to maintain thermodynamic consistency[18, 23], properties of infinite matter and coupled equations of motion for mesons, the values of nonlinear coupling constants are not free parameters to adjust at all. The more reliable values of observables and constraints are available, the better understanding of nuclear and astrophysical models will be certainly obtained.

The nonlinear interactions are constrained with self-consistency and thermodynamic consistency which generate rather constrained values of effective masses and vertex contributions, resulting in restricted properties of nuclear and neutron matter. A fine adjustment of nonlinear coupling constants is needed to produce the experimentally expected values of binding energy at saturation, $K$, $a_4$ and $M_{max}$. The mean-field approximation is applied to isospin-asymmetric matter; then, binding energies and densities, compressibility and symmetry energy at saturation are obtained. The results will be helpful to understand properties of neutron-rich nuclear matter and nuclei, as well as those of symmetric nuclear matter. Therefore, it is crucial to examine quantitatively the saturation properties of neutron-rich nuclear matter, such as the saturation density and binding energy, compressibility and symmetry energy of magic number neutron-rich nuclei. The density-dependent correlations of observables between isospin-symmetric and neutron-rich nuclei may be quantitatively examined by way of effective masses ($M^*$, $m^*_\omega$, $m^*_\pi$, $m^*_\rho$) and effective coupling constants ($g^*_\sigma$, $g^*_\omega$, $g^*_\rho$).

It is confirmed again that the effective masses of hadrons are essential for self-consistency to the approximation and to understand density-dependent correlations between properties of nuclear and neutron matter. The saturation properties of neutron-rich nuclear matter and high-density behavior of symmetry energy shown in the current approximation will not be reproduced without the isospin-asymmetry and the constraint of thermodynamic consistency. The properties of neutron-rich nuclear matter in the Table 3 are important results which should be examined by nuclear experimental data. The current mean-field approximation should be extended to HF, Ladder and other effective interactions by including hyperons[43, 44], and applications to astrophysical problems may yield interesting contributions to nuclear and astrophysics.

The nonlinear self-interactions of $g_{\sigma 4}$ and $g_{\omega 4}$, mixed-interaction of $g_{\sigma\omega}$ terms contribute significantly to determine the binding energy, compressibility and the mass of neutron stars. The nonlinear terms of $g_\rho$, $g_{\sigma\rho}$ and $g_{\omega\rho}$ yield significant contributions to symmetry energy by way of effective masses in the level of Hartree approximation. In addition, the density-dependences of effective coupling constants, $g^*_\sigma$, $g^*_\omega$ and $g^*_\rho$, appear to be important to reproduce compressibility $K$, symmetry energy $a_4$ and $M_{max}$ of neutron stars. However, since the effective masses and coupling constants are self-consistently related to one another by equations of motion and nonlinear coefficients, they have to be determined simultaneously so that equations of motion have solutions with conditions and constraints explained in sections. The existence of solutions to coupled equations of motion for mesons and convergences of properties of nuclear matter in the density range, $k_F = 1.0 \sim 3.0$ fm$^{-1}$, should be carefully checked so as to calculate properties of neutron stars and neutron-rich nuclear matter. The values of nonlinear coupling constants have to be interpreted as renormalized values of many-body interactions, which should be explained ultimately in terms of Hartree-Fock, Ring, Ladder and other effective approximations. The density-dependent correlations of effective masses and coupling constants should be investigated further in terms of many-body interactions.

A mixed-phase of quark-hadron neutron star might be expected[45]. A hadron-quark matter mixed-phase equation of state defined by MIT-bag model, for example, could produce the maximum observed mass: $M_{max} = 2.50\ M_{\odot}$ [46]. However, even if the experimental value of compressibility and the maximum mass of neutron stars are expected to be $K \sim 200$ MeV[47] and $M_{max} \sim 2.00\ M_{\odot}$, the current nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation indicates that it is not likely to generate hadronic neutron stars of the maximum mass of $M_{max} = 2.50\ M_{\odot}$. In order to distinguish the inner structure of neutron stars from the observed experimental data, it is important to analyze effective masses of baryons and mesons from nuclear experimental data as precisely as possible.

The current approach by employing the theory of conserving approximations is essential to check density-dependence of observables, self-consistency, Fermi-liquid properties and Landau parameters[16] of nuclear matter. It provides us with a method to construct a thermodynamically as well as microscopically consistent approximation, which is vital for applications to nuclear-astrophysical problems; on the contrary, the applications to high energy density objects might be used to examine the validity of nonlinear mean-field models of nuclear physics. The accumulation of data and the analysis of neutron stars, density-dependent correlations between nuclear and neutron stars should be investigated further in order to elucidate the validity of nonlinear mean-field models and theory of nuclear models.

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