Designing a Mechanism for Spectrum Trade toward Efficient Reallocation

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I. Introduction

This paper is concerned with spectrum pricing on the incentive-based behavior of spectrum users, and with a market mechanism (to be called extended market mechanism, EMM, in this paper) built on such pricing for spectrum reallocation.

The first part of this study begins with considering the behavior of incumbent and new users of spectrum with regard to spectrum pricing. A major economic incentive of spectrum users is to extract maximum returns from their activities. This paper states that the value of spectrum to incumbent users (the supply price of spectrum) and that to new users (the demand price for spectrum) are determined from their incentive-based behavior given business and other conditions. The study then proceeds to examining the trade and the welfare implications of such pricing. In particular, it is shown that in which case it is advantageous to transfer spectrum from incumbent to new users, and in which case such transfer will or will not take place in the form of spectrum trade as a consequence of their incentive-based behavior.

We further note that, in view of the history of spectrum use since the beginning of the 20th century, it is most likely that there remains extreme inefficiency in the use of spectrum today. In other words, it is possible for the society to be benefited greatly toward productivity increase and better life by reallocating spectrum being used inefficiently.

The second part of this study is devoted to proposing an economic mechanism for such reallocation (transfer) of spectrum. We first point out that, for efficient use of spectrum, it is necessary that both the demand price and the supply price be revealed truthfully; otherwise, it would be impossible to judge whether a particular transfer is, or is not, advantageous. We expect that, in the conventional market mechanism in which incumbent and new users meet and trade spectrum
voluntarily, the demand price of spectrum may be revealed truthfully as a consequence of economic incentives of new users; one way to bring this in is to conduct auction. There is no reason to expect, however, that incumbent users reveal the supply price in the market they participate voluntarily, since they can continue their business comfortably without trading spectrum while sitting on the rents earned from spectrum. That is to say, it is difficult to realize efficient transfer of spectrum by means of the conventional market mechanism.

To overcome this difficulty, the paper proposes EMM for spectrum transfer which will function, if implemented, roughly as stated below: First, each incumbent user of spectrum is obliged to reveal a supply price of his/her spectrum, that is, to declare a monetary amount, however large, for which the user is willing to yield the right of using the spectrum. Further, if an offer is made at a price greater than the declared supply price, then the user is obliged to transfer the spectrum to the party making the offer.

Second, each incumbent user pays to the government annually a spectrum-holding fee equal to the product of the declared supply price and the fee rate, the latter to be determined by the government. This is to prevent the user from claiming an unjustifiably high supply price; further, the fee may be regarded as a tax on using spectrum, which, after all, is the property of the society. Thus, in this mechanism, each spectrum user would either continue to use his/her spectrum with the annual payment, or cease using it for once-and-for-all compensation.

The objective of the second part of this paper is to examine the working of EMM at the conceptual level. Thus, we will show a model of each of the following: (1) incumbent users of spectrum with their business conditions, (2) potential users of spectrum with their behavior rules, and (3) expected outcome in spectrum reallocation. The model, however, is for explaining the working of EMM at large, not for describing in detail the working of EMM implemented for actual reallocation of spectrum.

In the model, we will postulate that incumbent and potential users follow their economic incentives. In other words, their behavior will be determined so as to maximize returns, which may be represented by the discounted sum of future returns from using (or not using) spectrum for their business. In particular, the prices bid or offered in EMM will be chosen according to their economic incentives, and the prices used for spectrum transfer in the model will be of incentive-based ones, within the framework of EMM.
II. Spectrum as an economic resource and its regulation—a short overview

A. What is spectrum?

We begin with considering physical and economic properties of spectrum. First of all, spectrum is a non-reproducible natural resource. It is different from oil or mineral deposits in that it does not deplete. It is different from produced capital like machines and equipment in that it does not depreciate. Spectrum, however, is a resource of limited supply with boundaries and a size.

In order to understand spectrum, it is useful to consider its resemblance to land as a resource. Land is a non-reproducible, non-depletable natural resource with limited supply; in addition, a piece of land has boundaries and a size. In fact, both land and spectrum as economic resources can be grouped into a category of space resources, of which examples are land space, water space, air space, the space of satellite orbits, to name a few. The resemblance of spectrum to land is a consequence of the fact that the utility of land arises from using a portion of the surface of the earth physically, whereas the utility of spectrum arises from using a portion of the surface of the earth electro-magnetically. Thus, the term “spectrum” means, in many cases, not electro-magnetic waves themselves, but a space for electro-magnetic waves to propagate through. In this paper, therefore, we will use the two terms “spectrum” and “spectrum space” interchangeably.

B. Division of spectrum into bands and blocks

A remarkable difference between land and spectrum is that, whereas there is only one piece of land attached to an area of the surface of the earth, there can be many spectrum spaces attached to such an area. In other words, spectrum space has an additional dimension, the frequencies of electro-magnetic wave. We state that land space is of two dimensions and spectrum space of three dimensions, of which two for the surface of the earth and the remaining one for the frequencies. In this paper, we use the term (spectrum) band to represent a segment of the axis measuring the frequencies. For example, the UHF television band may represent the frequencies ranging from 470MHz to 770MHz (width of 300MHz), and the band of television channel 20 may be the frequencies ranging from 512MHz to 518MHz (width of 6MHz). See Figure IIB.1.

We next define the term (spectrum) block to be a subset of the three-dimensional spectrum...
space composed of a band of frequencies and an area of land. See Figure IIB.2, in which is shown a spectrum block composed of an area $A$ of land and a frequency band $B$; such will be denoted as block $(B, A)$. Further, Figure IIB.3 illustrates ten spectrum blocks neighboring each other area-wise and/or frequency-wise, where, for simplicity, the three axes of the entire spectrum space are not shown explicitly in the figure. Note that Figure IIB.4 shows the six areas $A_1$ through $A_6$, and the two bands $B_1$ and $B_2$, used for composing the ten blocks in Figure IIB.3.

\begin{figure} [h]
\centering
\includegraphics[width=\textwidth]{figures}
\caption{Figures IIB.1 - 4}
\end{figure}

C. Regulation of spectrum use

We note that, in many countries, the utilization of radio spectrum is administered by government in two stages; the first is allocation of spectrum bands for specific objectives and the second is assignment of spectrum blocks to users.

Allocation of spectrum is done in two levels, international and national. International allocation of spectrum is conducted by the International Telecommunication Union (ITU) and by other international bodies. The national level of spectrum allocation is made by national government, which specifies one or more objectives for using a spectrum band in more detail together with technological specifications including the power of radio emissions, the allowance of interferences, the format of modulation and coding needed for information transmission, and others.

The second stage of managing spectrum is assignment of spectrum blocks to users. The entire spectrum space is divided into a number of spectrum blocks, to each of which a single user or multiple users are assigned with or without a license. An assignment also specifies the time (of a day, a week, etc.) in which spectrum is used together with detailed technological specifications.\footnote{Until this point, we have not mentioned of spectrum commons, an important mode of using spectrum. In the ordinary setting, commons is designated as a spectrum band which is open to the public for free use; no assignment is needed for commons. In our setting, however, for a reason to be explained later (→VIIA), spectrum commons is realized by assigning a block, which may be large area-wise, to a public agent (commons manager) and then having this agent let the block be used freely. Note that, in this setting, the agent can be regarded as one of spectrum users.}

III. Value of spectrum blocks to users

A. Theory of valuation (in general)

1. Value of economic objects

In order to consider how spectrum blocks are valued, we begin with the theory of valuation
applicable to economic objects in general. Examples of economic objects are household items such as furniture and other consumer durables, means of production such as machines and tools, real estate such as land and buildings, and business organization such as corporate firms. Any entity can be an economic object and can be valued as such as long as its ownership is established and transferable to others on some contract. In this paper, we are concerned only with valuation of economic objects in terms of money.

We next note that an economic object is valuable because it is useful to its owner. We distinguish two categories: in the first category, an economic object is valuable to its owner by itself for the reason that the object gives monetary returns to the owner. An example of this category is a financial asset such as a corporate share; its owner can expect to acquire a stream of dividends in the future.

In the second category, an economic object is valuable to the owner when it is used in combination with other objects or it is used as an integrated part of another entity; the value of such object depends on what other objects it is used with or what other entity it is used as an integrated part of. In short, the value of an object in this category depends on its environment in regards to the use of the object. Examples of such objects are a piece of furniture or consumer durables used in a particular household, machines or tools used for production in a particular firm, and real estates owned and used by a particular corporation. A spectrum block, when used to produce services such as mobile telephony or wireless Internet accesses, is of the second category. In particular, the value of a spectrum block depends on the environment in which it is used.

The property called (positive) externalities is remarkable with the second category of economic objects. Simply stated, externalities mean that the value of the entire object exceeds the sum of the values of the parts. For example, assume that the entire object is composed of parts $A$ and $B$. Let $F(A)$ and $F(B)$ be the value of $A$ and $B$, respectively, used separately, and $F(A, B)$ be their value used jointly as the entire object. Then externality means that

$$F(A, B) \geq F(A) + F(B).$$

(IIIA.1)

It is possible to write down inequalities such as this one for the case there are more than two parts composing the entire object. An extreme example is a productive firm (corporation) as the entire object, which is composed of managers, workers, and non-human productive resources, each of which may be regarded as a “part” of the corporation.
2. The supply and the demand prices of economic objects

The concept of the value of an economic object to its owner with economic incentives becomes useful when the owner contemplates changing the state of the object in question. We are interested here in two ways of changing the state of an economic object; one is to give up the ownership of an object currently in possession for receiving some amount of money (that is, selling the object), and the other is to acquire the ownership of an object currently possessed by some other in exchange for paying some amount of money (that is, buying the object). The value of an object to its current owner about to sell it is called its supply price, and the value of an object to its new and potential owner considering to buy it is called its demand price.

The concept of the supply and the demand prices is fundamental to the valuation of economic objects in general, and to the spectrum valuation in particular. Later, we will explain in detail how spectrum blocks are valued by current users and by new users. At this point, we only mention that the valuation of economic objects including spectrum blocks is done according to economic incentives; in other words, the supply and the demand prices are determined, and can therefore be explained by the behavior of current or new users seeking maximum economic returns.3

3. Evaluation of an economic object by means of its future returns

The value of an economic object in the first category is determined by the returns in the future from that object. In this case, the object is evaluated not in relation to other objects, nor as a part of another object; the value of the object is determined solely from its returns.

By the way, whether an economic object is evaluated as of the first, or of the second, category is determined mainly by the objective of the evaluation. For example, when a department of a large-sized corporation intends to acquire an economic object such as a spectrum block, the value of the department, not the value of the whole corporation, may be used to calculate the demand price for the object. An underlying assumption of such calculation may be that, the addition of the economic object (purchase of a spectrum block) will increase the value of the department, and the

3 The theory of valuation explained below in this paper may appear different from the textbook theory of prices based on the assumption of competitive market with supply and demand schedules (curves). The basis of the two theories are the same, though. Both of the two are constructed on the principle of economic incentives; the difference arises from the aspect each of the two concentrates upon. The textbook theory considers the value of an economic object varying in quantity, and emphasizes the valuation at the margin of quantitative changes. The theory presented here considers an economic object in an environment it is placed, and explains its value in relation to such environment; no quantitative change is taken into account, although the theory does not exclude the possibility of quantitative changes. In short, each of the two theories is a “view” from a particular aspect of a complicated entity: valuation of quantitatively changing economic objects in an economic environment considered explicitly.
value of the entire corporation will be increased by the same amount that the department value is increased. If the acquired object is considered to affect other departments of the corporation favorably, then the value of the department should be considered as of the second category and the value of the entire corporation would have to be considered to value the added object.

The value of an economic object of the first category may be obtained from its future returns in various ways including some subjective and discretionary judgment. There is a formula, however, widely used in accounting and economics, which is called the net present value (NPV), i.e., the discounted sum of returns in the future:

\[
\text{NPV} = \sum_{t=0}^{\infty} \left\{ \frac{1}{(1 + d)^{t-t_0}} \right\} \cdot R_t
\]

\[
F([R], d),
\]

\[
R_t \text{ denotes the return to the object during period } t,
\]

\[
\{R\} = \{R_1, R_2, \ldots\}, \text{ and}
\]

\[
d \text{ the discount factor (= interest rate plus risk-premium rate)}
\]

B. Spectrum valuation by incumbent users

We now proceed to explaining spectrum valuation as an application of the theory of valuation of economic objects both of the first and of the second categories. We first consider the valuation of spectrum by incumbent users facing possibility of selling spectrum blocks, i.e., the supply price of spectrum blocks.

1. Formula

Let us employ the following notations:

\[
P_X^1: \text{ the value of (incumbent) user X with a spectrum block}
\]

\[
P_X^2: \text{ the value of the user X without the block}
\]

Then, the value of the block to the user is expressed by the difference,

\[
P_X^* = P_X^1 - P_X^2.
\]
$P_1^X$ is the value of the operator $X$ with $B$ in the first category. Note that $P_1^X$ represents the total value of the returns to the operator $X$ in the future. $P_1^X$ may be calculated, as stated above, (1) by using the NPV formula with numerical forecasts of returns to $X$ in the future, or (2) by relying on the insights of experts/executives as to the expected performance of $X$ in the future, or (3) by the total value of the (corporate) stocks as exhibited in the stock market.

Suppose that $X$ loses the right of using $B$, and that the “best” alternative is to shift to using optical fibres instead. Then, $P_2^X$ is the value of $X$ without $B$ but with optical fibres obtained; $P_2^X$ may be calculated by means of the NPV formula.

To explain more in detail, observe that $P_2^X$ is the value of $X$ without $B$, but with all of the accompanying changes taken into account. When $X$ loses (sells) $B$, there may be several options that $X$ can take. One may be to simply decrease the scale of $X$’s operation by allowing some of $X$’s customers to leave. Another may be to keep the operation of $X$ at the same level as before by substituting some other means for spectrum (such as using optical fibres instead of spectrum or employing new technology for using the remaining spectrum more efficiently to cover the capacity of spectrum sold); in such cases, $X$ will need to spend some amount of money for the transfer to a new mode of business. For the case of shifting to optical fibers, the expenditure may include cost of fibres deployment, employee retraining cost, and cost of losing customers (and customer confidence) arising from the shift to fibres. $P_2^X$ is the value of $X$ without $B$, which is the sum of NPV of returns from $X$ without $B$ minus NPV of costs needed for $X$ to shift from its business with $B$ to that without $B$ (but with, e.g., optical fibres). If a set of values of $X$ without $B$ is calculated for more than one alternatives that $X$ may take after losing $B$, then, $P_2^X$ should be the maximum of such values, since $X$ would choose the alternative with the maximum value.

The value of $B$ to $X$ is of the second category, and equal to $P_1^X - P_2^X$.

The value of $B$ is the supply price of $B$, the lowest amount of compensation for which $X$ agrees to give up the use of $B$.

3. Expected behavior of incumbent users

   a. If a price above the supply price is offered for $B$, $X$ will “sell” $B$. The higher the price used for a sale, the better off $X$ will be.
b. If X is asked to exhibit a price for B, X can choose any price above the supply price. (In other words, X can tell a “lie” with regard to the “truthful” supply price of B.) The chances of successful sale, however, will be lower, the higher the price offered by X.

c. For a price lower than the supply price, X will never agree to give up B.

C. Spectrum valuation by potential users

1. Formula

Next, we consider the valuation of spectrum by new potential users, i.e., the demand price for spectrum blocks.

Let us use the following notations.

\[ P_{Y1} \]: the value of (potential) user Y without a spectrum block

\[ P_{Y2} \]: the value of the user with the block B

Then, the value of B to the user is expressed by the difference:

\[ P_{Y}^* = P_{Y2} - P_{Y1} \].

2. Example

Let a mobile operator (Y) intends to acquire spectrum block (B). In a typical case, Y has successfully developed new technology by means of which Y can provide new services (such as the 3rd generation mobile telephony) to customers if additional spectrum becomes available. Another case may be a wireless Internet service provider with new technology intending to expand its business by acquiring spectrum for this.

Note that \( P_{Y1} \) is the value of Y without B in the first category. \( P_{Y1} \) may be calculated by means of the NPV formula, or represented by the market value of Y’s stocks.

Suppose that Y obtains the right of using B, which will increase the value of Y. \( P_{Y2} \) is the value of Y with B in the first category; \( P_{Y2} \) may be obtained, say, by means of the NPV formula.

The value of B to Y is of the second category, and equal to \( P_{Y2} - P_{Y1} \), where \( P_{Y2} \) is the sum of discounted future returns to Y with B.

The value of B is the demand price for B, the greatest amount of money for which Y agrees to pay for obtaining the right of using B.

<Figure IIIC.1>

<Figure IIIC.2>
3. Expected behavior of potential users
   a. If B is available to Y at a price lower than the demand price, Y will “acquire” B. The lower the price used for such a trade, the better off Y will be.
   b. If Y is asked to reveal a price for B, then Y can choose any price lower than the demand price. (In other words, Y can tell a “lie” with regard to the “truthful” demand price for B.) The chances of successful purchase, however, will be lower, the lower the price bid by Y.
   c. For a price higher than the demand price, Y will never agree to pay for acquiring B.

4. Remark
   It is noted that the principle of deriving the demand price for spectrum as indicated above is analogous to that of deriving the supply price except that the direction of comparing the operator’s value is reversed. In other words, spectrum pricing is symmetrical between incumbent and new users at the theoretical level.

   In reality, of course, there is a great deal of difference between formation of a supply price and that of a demand price. A single most important factor may be the risk and uncertainty accompanying Y’s operation when it is of new category such as the case Y intends to use B to start a new service to customers with newly developed technology. In many cases, as we know well, returns from such venturing operation are uncertain; the demand price for B has to be formed with risk factors. In calculating NPV, such risk may be taken into account by increasing the discount rate. In short, therefore, the difficulty arising from the risk attached to future returns tends to lower the demand price for spectrum.

D. Trade implications of spectrum valuation
   In this and the following sections, we consider the implications of (truthful) valuation of spectrum block B on the possibility of trading B and the welfare of using B.

1. If \( P_Y^* > P_X^* \), and if X and Y can agree upon a price \( P \) such that \( P_Y^* \geq P \geq P_X^* \) for trading B from X to Y, then trade B will take place, since both the value of X and that of Y will be increased by such trade.
2. If $P^*_Y > P^*_X$ but X and Y cannot agree upon a price $P$ such that $P^*_Y \geq P \geq P^*_X$, then trade of B may not take place in spite of the possibility of welfare improvement to one or both of X and Y by trading B. There are a number of reasons of such no-trade outcome. First, X and Y may not meet at all in a decentralized market so that they never negotiate on price. Second, X and Y may meet in such a market, but each of them may continue offering prices which are not acceptable by the other so that negotiations end unsuccessfully. We discuss this issue in detail in one of the following sections (→V).

3. If $P^*_Y \leq P^*_X$, then there is no possibility of trading B from X to Y.

E. Welfare implications of spectrum valuation

In this section, we will consider the aggregate welfare (the welfare of X and Y summed up) as well as the individual welfares of X and Y. Let us first write down the notations:

1. Notations

- **B**: spectrum block
- $P^*_X$: the (truthful) supply price of B to incumbent user X.
- $P^*_Y$: the (truthful) demand price for B to potential user Y.

2. Aggregate welfare

   **Proposition**: If $P^*_Y > P^*_X$, that is, if the supply price exceeds the demand price for block B, then the aggregate welfare (the total value of X and Y added up) will be increased when B is transferred from X to Y by the amount equal to the difference $P^*_Y - P^*_X > 0$.

   **Proof**: We calculate the difference of the total value of X and Y before and after the transfer:

   \[
   (P^*_X + P^*_Y) - (P^*_Y + P^*_X) = (P^*_Y - P^*_Y) - (P^*_X - P^*_X) = P^*_Y - P^*_X > 0. \quad (\text{IIIE.1})
   \]

3. Individual welfares

   **Proposition**: Suppose that $P^*_Y > P^*_X$. If B is transferred to Y from X at a price $p$ such that $P^*_Y \geq p \geq P^*_X$, then at least one (and possibly both) of the value of X and that of Y will be increased and neither the value of X nor that of Y will be decreased.
Proof: To show this, we do the following calculations:

\[
\text{(the value of } X \text{ after the transfer plus income from the transfer)}
\]
\[
= P_X^2 + P \geq P_Y^2 + P_X^* = P_X^2 + (P_X^1 - P_X^2) = P_X^X
\]
\[
= (\text{the value of } X \text{ before the transfer}).
\]

\[
\text{(the value of } Y \text{ after the transfer minus transfer payment)}
\]
\[
= P_Y^2 - P \geq P_Y^2 - P_Y^* = P_Y^2 - (P_Y^1 - P_Y^2) = P_Y^Y
\]
\[
= (\text{the value of } Y \text{ before the transfer}). //
\]

In short, welfare improvement is achieved by a transfer of B from X to Y both at the individual and the aggregate levels (Pareto improvement). We note that if \( P_Y^* \leq P_X^* \), then there is no possibility of welfare improvement.

F. Measure of efficiency improvement of spectrum use

In view of the preceding discussion, we can devise a measure of efficiency improvement with regard to using block B and an aggregate measure of efficiency improvement with regard to a collection of blocks.

1. Measure of improvement of using a spectrum block

The following measure expresses the percentage increase in the total value of X and Y from transferring B such that \( P_X^* \geq P_Y^* \):

\[
\text{(measure of efficiency improvement with B)}
\]
\[
= 1.0 - \left( \frac{P_X^*}{P_Y^*} \right).
\]

(IIIE.4)

2. Aggregate measure of improvement of using a collection of blocks

Consider a collection of spectrum blocks. Examples of such a collection may be the UHF channels allocated for television broadcast, and the blocks in designated band assigned to wireless Internet access providers.

We first define

\[
\mathcal{P}_X = \sum P_X^*(B), \text{ and}
\]
\[
\mathcal{P}_Y = \sum \max[P_X^*(B), P_Y^*(B)],
\]

(IIIE.5) (IIIE.6)

where \( P_X^*(B) \) and \( P_Y^*(B) \) are the supply and the demand prices of block B, and the summation is taken over all B’s in the collection. Note that, for those B’s such that \( P_Y^* < P_X^* \), that is, those blocks with no possibility of improving spectrum use, we simply use \( P_X^* \) for \( P_Y^* \). Then,
\[ (aggregate \ measure \ of \ efficiency \ improvement) \]
\[ = 1.0 - \left( \frac{P_X}{P_Y} \right), \quad \text{(IIIE.7)} \]

which is the percentage increase in the sum of the value of the blocks in the collection.

Needless to say, if we attempt to calculate such measures of efficiency improvement, we need to devise some way to estimate \( P_X^*(B) \)'s and \( P_Y^*(B) \)'s, which is not an easy task. We will not get into this issue in this paper, though.

IV. Valuation of spectrum blocks with external economies

A. Introduction

A remarkable property of spectrum resources, as of other space resources, is the presence of external economies. It is more advantageous to use more than one blocks of spectrum located near-by area-wise and/or frequency-wise jointly than to use each of them separately; this is also called (positive) externalities in using spectrum. For the case of land, an example of economies of scale may be seen in the traffic capacity of highways; the capacity of a two-lane highway is greater than twice of that of a single-lane one, and the capacity of a three-lane highway is greater than one-and-half times of that of a two-lane one. Similar examples may be found with land space used for buildings. For the case of spectrum, an example is the fact that communications channels (such as television broadcasting channels) are assigned in near-by frequencies (such as in UHF bands) to contribute cost saving in manufacturing television receivers. Further, the benefit obtained by sharing spectrum by means of spread-spectrum technology is an example of external economies in spectrum utilization.

In short, we can state that the greater the size of spectrum blocks, the higher the technological efficiency of spectrum utilization. If we push this reasoning to an extreme, the most efficient way of using spectrum would be complete centralization of spectrum management. In such a world, all spectrum blocks would be integrated into a huge single spectrum block, which would be managed by a central authority to exploit the benefit of external economies as much as possible.

Actually, however, there are other reasons to favor decentralized management of spectrum. One is the fact that spectrum is used for various purposes; because of the efficiency of “division of labor,” spectrum should be divided into bands and blocks, as it actually is, according to the purpose of utilization, each of which is to be managed in a decentralized way. Another reason is the need for using the power of competition and free market, since competition presupposes decentralization.
of spectrum management and/or its use.

Thus, in considering spectrum management and also in dealing with the economics of spectrum use, we have to take into account two “forces” working in opposite directions. One is external economies calling for centralization, and the other is a number of other factors calling for decentralization, of spectrum use. Because of this, one cannot avoid some “compromise at a middle point”, and such compromise makes it almost impossible to form a consistent policy, or a systematic theory, to deal with the issue of efficient use of spectrum.

In this section, in view of the observations above, we consider externalities only to a limited extent; a full-scale study of externalities is left for research in the future. Specifically, we consider, in this section, the valuation of spectrum blocks by incumbent and potential users in the presence of external economies. The reader is warned that, for this reason, the present section is parenthetical in this paper.

B. Representation of externalities

1. Notations

We begin with writing down the notations to be used in this section. When external economies exist, the value of a spectrum block depends on the way it is used with other blocks. We will therefore need to consider not only individual blocks, but also collections of blocks, which we call groups in this section. Thus, let a group of spectrum blocks be one or more spectrum blocks used jointly by one incumbent or potential user. Let individual blocks be denoted by A, B, C, …, and groups (of blocks) by AB, ABC, CD, …, where AB means that two blocks A and B are used jointly, ABC means that three blocks A, B, and C are used jointly, and so on.

It is understood that, whenever we write AB or ABC, the two blocks A and B or the three blocks A, B, and C are “neighbors” area-wise and/or frequency-wise, so that it makes sense to join blocks into groups.

Let the value of individual blocks and that of groups be denoted by F(A), F(B), F(AB), …, where F(A) is the value of A used by itself, F(B) is the value of B used by itself, and F(AB) is the value of the group A and B used jointly.

2. External economies

By using the notations introduced above, we can express external economies as a property of
the value function $F$: Let $A$, $B$, and $AB$ be blocks or groups as defined above. Then, the value function $F$ has the property of external economies, when

$$F(A) + F(B) \leq F(AB),$$

(IVB.1)

where $AB$ is the group formed by $A$ and $B$; i.e., the value function is convex with regard to the formation of groups over individual blocks/groups with neighboring relations.

C. Examples

In this subsection, we show examples of spectrum blocks and groups with externalities. First, Figure IVC.1 is an example of groups formed on nine blocks $A$, $B$, …, $I$. Note that all of the blocks are in neighboring relations directly or indirectly; in other words, all of the blocks can be “connected” by means of neighboring relations. A simple case is the one in which blocks $A$ through $I$ are “connected” linearly in that order with $A$ and $I$ as “end blocks.” Note that there may be other “connections” in addition to the linear one. Note further that Figure IVC.1 is but one of the possible groups which can be formed on the nine blocks with given neighboring relations. Thus, we can consider a group as a tree in which each leaf is an individual spectrum block, and each node is a join of leaves or nodes. A tree represents a way in which spectrum blocks are joined in view of given external economies. The number of such trees which can be formed on given blocks with given neighboring relations may be large, but is finite as long as the number of underlying blocks is finite.

<Figure IVC.1>

Next, Figures IVC.2 and 3 are examples of valuation of blocks and groups with externalities. In Figure IVC.2, the value of each of the blocks $A$ and $B$, used separately, is equal to 5, whereas the two blocks used jointly is 15, which is greater than the sum ($= 5 + 5 = 10$) of the values of the two blocks used separately: $F(AB) > F(A) + F(B)$. In this case, if an incumbent is using blocks $A$ and $B$ jointly, then the (truthful) supply price of $AB$ is 15, but the (truthful) supply price of $A$ or $B$ used by itself will be 10 ($= 15 - 5$), since the incumbent with $AB$ of value 15 would agree to lose $A$ and to keep $B$ only if he/she is compensated by 10 for $A$ to recover the value 15 of $AB$. In Figure IVC.3 with three blocks $A$, $B$, and $C$ and groups $AB$ and $ABC$, the (truthful) supply prices of $A$, $B$, $C$, and $AB$ will be as follows:
D. Implications of externalities

From the examples shown above, one can see that introducing externalities to the use of blocks will greatly increase the domain of objects for valuation, if no restriction is imposed on the formation of groups (trees). One of its implications is that transactions cost of valuation of spectrum and that of spectrum trade would be high with externalities, although the use of computer technology may make it possible to handle such transactions without increasing transactions cost to a prohibitive level.

Further, we observe that the introduction of externalities will affect both incumbent and potential users symmetrically. However, externalities may increase the asymmetry between incumbent and potential users arising from other reasons (see VG).

In the rest of the paper, we will assume that no externality exists with spectrum blocks, leaving this issue for research in the future.

V. Conventional market mechanism for spectrum trade

A. Introduction

The issue of efficient use of spectrum is how to designate a user to each spectrum block so as to maximize the total return from the entire spectrum resources. A textbook answer is that, if spectrum is freely traded, then each block will be held, at least in the long run, by the user who can offer the highest price for it, that is, the user who can use it most efficiently, leading to an efficient use of the spectrum blocks. This would be true if the spectrum block were an ordinary commodity like foods or clothing for which substitutes are available in the market and competition exists both on the demand side and the supply side of the market. Actually, however, the spectrum block, as an

<table>
<thead>
<tr>
<th>blocks/groups</th>
<th>(truthful) supply price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$15 = 25 - (5 + 5)$</td>
</tr>
<tr>
<td>B</td>
<td>$15 = 25 - (5 + 5)$</td>
</tr>
<tr>
<td>C</td>
<td>$10 = 25 - 15$</td>
</tr>
<tr>
<td>AB</td>
<td>$20 = 25 - 5$</td>
</tr>
<tr>
<td>ABC</td>
<td>$25$</td>
</tr>
</tbody>
</table>

<Figure IVC.2>

<Figure IVC.3>
object for trade, is quite different from ordinary commodities; we cannot expect that mere introduction of spectrum trade will lead to its efficient use.

In fact, a spectrum block is characterized by its band and location (see IIB); as such, it is a unique entity and there is no perfect substitute for it. In fact, a spectrum user is a local monopolist of the block he/she is using. In addition, a spectrum user has vested rights with his/her block because of investments made on it in the past, most of which became sunk cost (unrecoverable cost); for a mobile telephone operator, the cost spent on communications equipment and that spent to obtain the customers are examples of sunk costs.

Furthermore, as we saw in the last section, we saw that the presence of external economies in using spectrum blocks would greatly increase, when compared with a case without externalities, the size of the domain of pricing for users; because of this, transactions cost would be higher, and the speed of market adjustment would be slower, in trading spectrum with externalities than without.

In this section, we will look into this issue in detail, except that we assume away externalities, by considering the behavior of incumbent and potential users of spectrum and the functioning of conventional market mechanism for spectrum trade.

B. Outline of conventional market mechanism

First of all, we summarize what is meant by conventional market mechanism for spectrum trade. In short, it means the outcome we would see when the spectrum blocks became a privately-owned property and a user-owner of a spectrum block could sell it to another user-owner freely at an agreed-upon price, where all regulations in using spectrum are to be observed by users equally before and after trade except, of course, those regulations concerning spectrum assignments. Thus, we can consider spectrum trade in conventional market mechanism as free and voluntary trade of spectrum licenses.

The following summarizes the way such a market works:
In the following, we examine the functioning of such market by constructing a model of it. In particular, we will be concerned mainly with the outcome of the bargaining conducted by a pair of an incumbent user and a potential user on a particular spectrum block.

For simplicity, we will assume away the presence of external economies in using spectrum block in this and the following sections.

C. Notations

In this section, we present a model of conventional market mechanism using the following notations. Note that we will distinguish various prices of a spectrum block carefully; they include truthful supply and demand prices, prices offered or bid in bargaining, and prices used for actual trade of spectrum.

1. spectrum blocks and users:
   - spectrum block: B
   - incumbent (current) user of B: X
   - potential (new) user of B: Y

2. prices derived from spectrum valuation (truthful prices):
   - supply price of B by X: $P_X^*$
   - demand price for B by Y: $P_Y^*$

3. block prices used in bargaining:
   - price offered for B by X: $P_X$
   - price bid for B by Y: $P_Y$

4. block prices used in actual trade:
price received for B by X: $\tilde{P}_X$
price paid for B by Y: $\tilde{P}_Y$

5. bargaining periods: $t$
   
   $t = 0$: initial period
   $t = \tilde{t}$: final period reached
   $\tilde{t}_X$: maximum bargaining period for X
   $\tilde{t}_Y$: maximum bargaining period for Y

D. Overall view of trade possibilities

We begin with considering a pair of an incumbent user X with block B and a potential user Y interested in obtaining the block B. Suppose that X and Y meet in the market and start negotiating on possible trade of B. Let us assume that

$$P_X^* \leq P_Y^*,$$
(VD.1)

for, if inequality (VD.1) is not satisfied, there would be no possibility of trade, and the pair will be dissolved sooner or later after some bargaining.

To show the possibility of trading B between X and Y given $(P_X^*, P_Y^*)$ satisfying (VD.1), we present a graph with the horizontal axis for $P_X$ and the vertical axis for $P_Y$ as in Figure VD.1. Then, the range $TR$ of successful trade prices $(\tilde{P}_X, \tilde{P}_Y)$ is defined as follows:

$$TR = TR(P_X^*, P_Y^*) = \{(\tilde{P}_X, \tilde{P}_Y); P_X^* \leq \tilde{P}_X \leq \tilde{P}_Y \leq P_Y^*\},$$
(VD.2)

which is the triangle $\Delta WUV$ in Figure VD.1.

Note that, in Figure VD.1, a price $\tilde{P}_X$ lower than $P_X^*$ is not acceptable to X and a price $\tilde{P}_Y$ greater than $P_Y^*$ is not acceptable to Y, so that the shaded area to the left of line WU and that to the above of line WV are excluded from trade possibilities. Further, in order for B to be traded, the price $\tilde{P}_Y$ to be paid by Y must at least be as high as the price $\tilde{P}_X$ to be received by X; hence, the shaded area to the right-lower side of the 45-degree-line UV is also excluded from trade possibilities. Triangle $\Delta WUV$ is the set of price pairs $(\tilde{P}_X, \tilde{P}_Y)$ left out in the graph after these exclusions.

Observe that, if $P_X^* > P_Y^*$ violating inequality (VD.1), then the triangle cannot be shown (all price pairs are excluded in the graph).

Observe further the following:
1. If actual trade takes place with a price pair \((\tilde{P}_X, \tilde{P}_Y)\) on the segment UV, then the price paid by Y is equal to the price received by X; all of the surplus \((\tilde{P}_Y - \tilde{P}_X > 0)\) of this trade is taken by X and Y. How the surplus is divided between X and Y is determined by at which point on the segment UV is chosen for the trade; in reality, this will depend on the way in which bargaining is conducted between X and Y. We will consider this point in more detail in the last subsection of this section.

2. If actual trade takes place with the price pair \((\tilde{P}_X, \tilde{P}_Y)\), which is represented by point W in the graph, then the price paid by Y is greater than the price received by X by the amount equal to the entire surplus of block B, \(\tilde{P}_Y - \tilde{P}_X > 0\).

Where this surplus goes to depends on institutional settings. A common example is that the surplus goes to the government in the form of, say, sales tax or auction “tax.”

3. If the price \((\tilde{P}_X, \tilde{P}_Y)\) for actual trade is represented by a point inside the triangle \(\Delta WUV\), then the surplus will go to X, Y and a third party (e.g., the government) each in part. How it is divided into the three depends on bargaining between X and Y and institutional settings.

E. The behavior of incumbent and potential users—bilateral bargaining

1. Overview

In this section, we consider how a pair of an incumbent user and a potential user, who meet randomly in the conventional market, will negotiate for possible trade of block B. First of all, we note that there is no reason for a user to reveal the supply or the demand price truthfully; on the contrary, we expect that, in most cases, the incumbent will offer a price, say \(P_X\), greater than the supply price \(P_X^*\), and the potential user will bid a price, say \(P_Y\), lower than the demand price \(P_Y^*\), to seek extra profits from trading B. If it happens that \(P_X \leq P_Y\), then the two will trade B at a price, say \(\tilde{P}\), such that \(P_X \leq \tilde{P} \leq P_Y\), and the bargaining will end at that point. If it holds that \(P_X > P_Y\), then this means that the two have not found a price acceptable to each of them. They may then continue negotiation by revising their prices, seeking possible trade of B, or else they may stop negotiation at that point giving up trading B. Thus, outcome from such bilateral bargaining is determined by the behavior of the two concerning whether, and how, they revise prices in their negotiation.

In the following, we will spell out what is stated above in the form of a model.
2. Incumbent users

Let $t$ denote time period for bargaining:

$$t = 0, 1, 2, \ldots,$$

(VE.1)

so that $P_X(t)$ is the price offered by $X$ for selling $B$ at period $t$; in particular, $P_X(0)$ is the initial offer.

Then, the behavior of $X$ in the bargaining may be expressed by $P_X(t), t = 0, 1, \ldots$, such that

$$P_X(t) \leq P_X^*, \text{ for all } t,$$

(VE.2)

and

$$P_X(0) \geq P_X(1) \geq P_X(2) \geq \cdots$$

(VE.3)

In period $t$, $X$ chooses a price $P_X(t)$ satisfying inequalities (VE.2 and 3). The information that $X$ has in that period is the supply price $P_X^*$ and all the prices offered or bid during the preceding periods, $s < t$. In other words, the information $X$ has in period $t$ is

$$I_X(t) = \{P_X; (P_X(s), P_Y(s)), s = 0, 1, 2, \ldots, t - 1\}. $$

(VE.4)

The behavior of $X$ in choosing $P_X(t)$ in period $t$ may be specified by a function, say $G_X$, of $I_X(t)$ into a set, say $J_X(t)$:

$$G_X: I_X(t) \mapsto J_X(t),$$

(VE.5)

where

$$J_X(t) = \{P: P_X(t - 1) \geq P \geq P_X^*\}. $$

(VE.6)

In this paper, we will not attempt to specify the actual form of $G_X$. Figure VE.1 illustrate the behavior of $X$ in $t$, where $X$ chooses a price $P_X(t)$ on the horizontal axis, with the information of the supply price $P_X^*$ and the history of bargaining $(0 \leq s \leq t - 1)$ depicted by the curve connecting $Z(0) = (P_X(0), P_Y(0))$ and $Z(t - 1) = (P_X(t - 1), P_Y(t - 1)).$

3. Potential users

The adjustment behavior of $Y$ in bargaining is basically similar to that of $X$; it may be expressed by $P_Y(t), t = 0, 1, \ldots$, such that

$$P_Y(t) \leq P_Y^*, \text{ for all } t,$$

(VE.7)

$$P_Y(0) \leq P_Y(1) \leq P_Y(2) \leq \cdots$$

(VE.8)

In period $t$, $Y$ chooses a price $P_Y(t)$ satisfying inequalities (VE.7 and 8) with the information of the demand price $P_Y^*$ and all the prices offered or bid during the preceding periods, $s < t$. That is,
the information $Y$ has in period $t$ is

$$I_Y(t) = \{P_Y^s; (P_X(s), P_Y(s)), s = 0,1,2, ..., t-1\}. \tag{VE.9}$$

The function $G_Y$ expressing the behavior of $Y$ in period $t$ is:

$$G_Y: I_Y(t) \mapsto J_Y(t), \tag{VE.10}$$

where

$$J_Y(t) = \{P: P_Y(t-1) \leq P \leq P_Y^s\}. \tag{VE.11}$$

Figure VE.2 illustrates the behavior of $Y$, where $Y$ chooses a price $P_Y(t)$ on the vertical axis, with the information of the demand price $P_Y^s$ and the history of bargaining ($0 \leq s \leq t-1$) depicted by the curve connecting $Z(0)$ and $Z(t-1)$.

<Figure VE.2>

4. Stopping rules

In this model, the bargaining between $X$ and $Y$ may stop for two reasons. The first is the case of successful bargaining; that is, in period $t$, a price pair $Z(t) = (P_X(t), P_Y(t))$ satisfying the trade condition:

$$P_X^* \leq P_X(t) \leq P_Y(t) \leq P_Y^* \tag{VE.12}$$

is reached. In this case, the bargaining is ended and block $B$ will be traded from $X$ to $Y$ for a price $\tilde{P}$ satisfying

$$P_X(t) \leq \tilde{P} \leq P_Y(t). \tag{VE.13}$$

Both $X$ and $Y$ will leave the market with regard to trade of $B$.

The second is the case of unsuccessful bargaining. Let $\bar{t}_X$ and $\bar{t}_Y$ be the maximum period of bargaining for $X$ and $Y$, respectively. If the maximum-period constraint

$$t \leq \bar{t}_X \text{ or } t \leq \bar{t}_Y \tag{VE.14}$$

is satisfied with equality with some period $t$, the bargaining will stop. In this case, block $B$ will not be traded and will continue to be held by the incumbent $X$. $X$ and/or $Y$ may stay in the market for other possibilities of spectrum trade, or may leave the market.

F. Summary

Figure VF.1 illustrates a case of successful bargaining on a block $B$ by an incumbent $X$ and a potential user $Y$. It starts at $Z(0)$ in the initial period $t = 0$. The prices bid or offered successively for $t = 1,2, ...$ are shown by the curve starting at $Z(0)$. In this case, the bargaining ends in
period $t = \bar{t}$, where $Z(\bar{t})$ is in the triangle $\Delta WUV$. An example of unsuccessful bargaining may be given by a case with which the curve ends at a point, say $Z(\bar{t}')$, outside of the triangle.

Thus, we can conclude that whether a bargaining will or will not be successful is determined by the following factors:

1. the initial prices: $Z(0) = (P_X(0), P_Y(0))$.
2. the maximum periods for bargaining: $\bar{t}_X$ and $\bar{t}_Y$.
3. the speed in which prices offered or bid are revised in bargaining, which will depend on the functions $G^X$ and $G^Y$.

The process of a bargaining expressed in a form of algorithm:

```
i. let $t = 0$
   set $P_X(0), P_Y(0)$: initial prices
ii. (main bargaining process)
   while $P_X(t) > P_Y(t)$, repeat a., b., and c.:
   a. revise $P_X(t)$ and $P_Y(t)$ into $P_X(t + 1)$ and $P_Y(t + 1)$, respectively;
   b. if $t \geq \bar{t}_X$ or $t \geq \bar{t}_Y$, then go to 4;
   c. let $t \leftarrow t + 1$.
iii. (successful trade prices reached: $P_X(t) \leq P_Y(t)$)
   choose $\tilde{P}_X$ and $\tilde{P}_Y$ such that $P_X(t) \leq \tilde{P}_X \leq \tilde{P}_Y \leq P_Y(t)$ according to a predetermined rule;
   go to 5.
iv. (maximum trade period reached, no trade)
v. let $\bar{t} = t$;
end.
```

G. Business conditions of incumbent and potential users

In this subsection, we consider the effectiveness of the conventional market for spectrum trade in reallocating spectrum blocks which can be used more efficiently by new potential users than by incumbents. Specifically, we are concerned with the factors which determine whether a bargaining between an incumbent and a potential user will, or will not, end successfully. This means that we
examine the items 1 - 3 listed in subsection VE. In this paper we will do this by considering the business conditions of incumbents and potential users of spectrum in an intuitive, non-rigorous way.

First, we consider an incumbent user (X), running a business by using spectrum block B. We note that X is a local monopoly in using B, enjoying extra profits from the monopoly rent of B (see VA). Thus, we expect that, in average, the business condition of incumbents is good on accumulated profits; that is to say, there is no urgent need for them to sell B. In short, incumbents can continue to sit on B comfortably.

The business condition of potential user (Y) is quite different from that of incumbents. We note that, in many cases, potential users are creating new business for starting new services by using B. For this reason, it is likely that potential users have few extra profits accumulated; on the contrary, they likely need to generate profits urgently for paying interests and/or dividends on newly raised capital. To satisfy this, incumbents will urgently need to obtain B, in order to run their business to generate profits.

Based on the observations above, we state the following assertions:

1. on maximum trade periods:

   The maximum bargaining period of X ($\tilde{t}_X$) will be greater than that of Y ($\tilde{t}_Y$), since Y cannot wait for long as X can.

   $\tilde{t}_X > \tilde{t}_Y$.  

2. on the rate of revising prices offered/bid:

   Let the rate of revising prices by X and Y be defined respectively by

   $\hat{p}_X(t) = \frac{|p_X(t) - p_X(t-1)|}{p_X(t-1)}$, and

   $\hat{p}_Y(t) = \frac{|p_Y(t) - p_Y(t-1)|}{p_Y(t-1)}$.  

   Then, we assert that the rate $\hat{p}_Y$ will be far greater than the rate $\hat{p}_X$, since it is likely that Y attempts to revise its price at a high rate with the hope that such revision brings $p_Y$ within the range of prices for which X will agree to trade B. Thus,

   $\hat{p}_X(t) < \hat{p}_Y(t)$;  

   Y revises prices faster than X does.

Figure VG.1 gives an example of an unsuccessful bargaining, which is a typical and majority case in the conventional market of which the business conditions of incumbent and potential users are as stated above. A curve representing such a bargaining starts at $Z(0)$, which is located to the
south-east to point $V$ of the triangle, then proceeds mostly upward throughout the bargaining periods, since $X$ will not revise its price much in each period of the bargaining, and ends at point $Z(\bar{t})$, which is located outside of the triangle, since $Y$ cannot wait for long and will finish (i.e., give up) the negotiation, an unsuccessful ending.

H. **Expected outcome from conventional market mechanism**

Finally in this section, we summarize the effects of conventional market mechanism reallocating spectrum for improving the overall efficiency of spectrum use. We do this by presenting a scenario for the case in which no spectrum reallocation was attempted until time $t_0$, and conventional market mechanism was introduced at time $t_0$ and thereafter.

Let

$$P_Y(t) = \sum P^*_Y(B,t), \quad \text{and}$$

$$P_X(t) = \sum P^*_X(B,t)$$

(VH.1)

(VH.2)

where $P^*_Y(B,t)$ and $P^*_X(B,t)$ are the demand and the supply prices of block $B$ at time $t$, and the summation is taken over all spectrum blocks in question. $P_Y(t)$ and $P_X(t)$ may be regarded as the potential and the actual values of the spectrum blocks in question. In other words, $P_Y(t)$ is the value when the spectrum blocks are used most efficiently with given technology of using spectrum.

We consider two cases with regard to the presence of technological progress. First, we assume that there is no technological progress and, hence, the overall state of the economy is stationary:

$$P_Y(t) = \bar{P}_Y = \text{const.}, \quad \text{for all } t.$$  

(VH.3)

Suppose further that $P_X(t) < \bar{P}_Y$ for $t < t_0$, since there was no reallocation of spectrum until $t = t_0$, and the actual value of spectrum was below its potential value.

Suppose that, at $t = t_0$, conventional market mechanism was introduced and the users are allowed to trade freely. Then, spectrum trade will start taking place and $P_X(t)$ will start increasing. Because of the rigidity of the conventional market mechanism as discussed above, however, the speed of increase in $P_X(t)$ may be slow; $P_X(t)$ may get to be close to $\bar{P}_Y$ years or decades after $t = t_0$. See Figure VH.1.

Next, we assume that technology for using spectrum is progressing at some pace throughout. Consequently, the potential value $P_Y(t)$ increases at some speed. See Figure VH.2. The actual value $P_X(t)$ will increase, as in the previous case, after $t_0$, the time the conventional market
mechanism was introduced. Whether the “distance” between $P_Y(t)$ and $P_X(t)$ will be narrowed or widened depends on the speed of technological progress and the speed of the improvement of the efficiency in spectrum use with the conventional market mechanism.

VI. Extended market mechanism (EMM) for spectrum trade

A. Outline of extended market mechanism

We have just found that conventional market mechanism, which allows incumbent and potential users to trade freely but voluntarily, will not function well in reallocating spectrum for efficient use. In this section, we propose extended market mechanism (EMM) which can facilitate spectrum trade toward a Pareto-optimal state.

In order to introduce extended market mechanism for spectrum trade, we first propose to divide spectrum regulations into two parts, say, part A and part B. See Figure VIA.1. In short, part B is concerned with determining who uses each spectrum block. Part A deals with everything else; it is composed mainly of engineering and technological regulations. Thus, in part A, a government agency determines how to divide the frequencies into bands and how to divide the spectrum space into blocks with technological details. In short, Part A is concerned with “for what purpose(s)” and “how” each spectrum block is used. Part B determines “by whom” only.

The objective of the following is to design and analyze an economic model (extended market mechanism) for part B; we will not deal with part A in this paper. We point out, however, that information generated in part B such as prices used for spectrum trade should be useful for regulations in part A.

Observe that EMM should at least be able to have each incumbent reveal the supply price of his/her spectrum truthfully, since, without truthful supply prices, it is impossible to judge whether a particular trade is, or is not, Pareto-improving. Further, EMM should also be able to centralize the information about (truthful) supply prices so that each potential user can find, if any, an incumbent with whom a Pareto-improving trade may be sought. In other words, EMM should be an organized market, in which information pertinent to Pareto-improving trade is collected and distributed systematically. For this, we need a market operator (auctioneer).

Figure VIA.2 outlines such EMM. The supply side of EMM is the incumbent users (X) of

---

4 An example of organized (centralized) market is stock market. Note, however, that for stock market one need not worry about the possibility of untruthful price revelation.
spectrum blocks. Each incumbent is obliged to reveal the supply price ($C$) of his/her spectrum truthfully, and the spectrum holding fee is imposed to incumbents for this.

Potential users (Y) of spectrum blocks can access, if they so choose, to the supply price of each block. Further, if Y wishes to obtain the right to use a block, Y can enter into an auction for it. Y will actually acquire the spectrum if he/she wins the auction.

Finally, Government (Z) operates the market as an auctioneer as well as the spectrum manager. Z specifies a spectrum-holding fee rate ($r$), and collects spectrum fees ($R$) from each incumbent. In addition, Z oversees an auction for each spectrum block with one or more potential users seeking the right of using it. If a winning bid (the highest demand price) exceeds the supply price of a block, Z receives the difference (surplus) between the demand and the supply prices. Both spectrum holding fees and auction surplus should be treated as a property tax on spectrum by the government.

<Figure VIA.1>
<Figure VIA.2>

**B. Bill of spectrum rights and responsibilities (proposed)**

Introducing the framework of EMM as explained above means that the rights and the responsibilities of spectrum users, both incumbent and potential, would be changed in EMM from the ones prevailing currently. In this regard, we have to point out that the rights and the responsibilities of spectrum users are unclear, or stated explicitly but in an abstract form, in the current system of spectrum use.

For instance, it is customary that spectrum law requires spectrum to be used for the benefit of the society, not for the benefit of individuals or business firms. This means that a block of spectrum being used by an incumbent inefficiently for the society should be reallocated to some other user, if such exists, who could use it more efficiently than the incumbent. Yet we observe that such reallocation is quite rare; most of the inefficient users of spectrum are allowed to keep their licenses/concessions continuously. In effect, incumbents hold vested rights on spectrum.

It is clear that EMM would change such vested rights of incumbents. In this sense, introduction of EMM implies introduction of new rights and responsibilities to spectrum users. It is not the purpose of this paper to write down a comprehensive “spectrum bill.” In the following, therefore, we write down a “bill of spectrum rights and responsibilities (proposed)” in a simple form, that is, only to the extent implied by EMM.
C. Rights and responsibilities of incumbents

In the following, we propose EMM by specifying the rights and responsibilities of its participants: incumbents (current users), new users (potential users), and the government. We begin with the incumbents.

As stated in the preceding subsection, spectrum is a property of the society as a whole. The right to use a spectrum block is given to a user with the following responsibilities:

1. Each user shall reveal the supply price \( C \) of the block, where \( C \) is the least amount of compensation for which the incumbent agrees to yield the right of using the block.
2. Each user shall pay the spectrum usage fee \( R = r \cdot C \), where \( r \) is the (annual) rate of spectrum usage fee to be determined by the government.

Thus, the incumbent may continue using a block if there is no offer greater than the supply price \( C \), and he/she must yield the block if there is an offer greater than \( C \).

Under such a regime, incumbents tend to declare a high \( C \) for continuing the use of a block on the one hand, but have incentive to keep \( C \) low for saving the payment \( R \) on the other. This means that there is a tradeoff to incumbents; therefore, “holding up” a block may be costly to them.

We propose here that “incumbents” should include all users of spectrum, regardless whether they are individual, business, or government users.

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Bill of Spectrum Rights and Responsibilities:

1. Spectrum is a property owned by the people collectively; the benefits of using, and the income from operating, spectrum shall therefore be attributed to the people.
2. Spectrum may be used exclusively by a user for an indefinite period; the right to use spectrum, however, is by no means permanent.
3. The user shall yield the right of using spectrum when requested by a party with compensation which exceeds the amount declared by the user prior to such a request.
4. The user shall pay each year to the government a usage fee, which is equal to the product of the declared compensation and a fee rate to be specified by the government.
Observe that, to incumbent users, the system EMM is a mandatory insurance with compensation. For, each user declares a monetary amount of compensation to be paid to the user in the event that the spectrum block becomes unusable. That is, each user pays an insurance premium (spectrum holding fee) to the government annually, which is equal to the amount of compensation declared (the supply price revealed) multiplied by the fee rate to be determined by the government. In other words, the system can be considered as a casualty insurance plan, where a casualty here is the event that the spectrum block becomes unusable.\(^5\)

We further note that, given a fair insurance program in which the premium rate \((r)\) is equal to the probability \((\pi)\) of the event that spectrum trade takes place, a rational risk-averse user will choose a complete insurance plan in which the utility in the event of trade be equal to the utility in the event of no trade.\(^6\)

We leave it for research in the future to consider the short- and long-run behavior of spectrum users and its equilibrium and welfare implications.

Observe that the position of an incumbent in the proposed EMM is quite different from that of an incumbent in the conventional market mechanism in which the possibility of spectrum trade is introduced on top of the traditional licensing system. We note that, in the latter, the incumbent’s right of using spectrum is protected heavily. There are two factors which limit this: one is the case that the incumbent’s license is denied from renewal, and the other is the case that the spectrum usage fee imposed on the incumbent becomes too high for him/her to continue using it. Otherwise, the incumbent can continue using spectrum even when there are other users who can use it more efficiently.

In EMM, the incumbent is exposed to competition by potential users; he/she has to yield the right of using spectrum to a potential user who can use it more efficiently for a compensation specified by the incumbent himself/herself. We can say that EMM would replace the system of administrative licensing and usage fees into an economic system of the (self-revealed) supply price and the (self-chosen) usage fee.

\(^{5}\) Ikeda and Ye [2003] proposed a system of “reverse auction” for spectrum reallocation, in which the supply price of spectrum is revealed by incumbent users who attempt to “sell” the right to use spectrum to the government at a price as high as possible. Their system, however, does not have a provision of insurance and compensation.

\(^{6}\) See, e.g., Mas-Colell, et al. [1995], pp.187-188.
Incumbent Users in EMM:

1. Revelation of supply price \( (C) \) of each block
   \[ C: \] the least amount of compensation for which incumbent agrees to yield the right of using the block
2. Payment of spectrum usage fee \((R)\)
   \[ R = r \cdot C. \]
   \[ r: \] (annual) rate of spectrum usage fee to be determined by the government
3. Incumbents
   may continue using a group if there is no offer \( > C \)
   must yield the block if there is an offer \( \geq C \)
4. Determination of \( C \) by incumbents:
   Incumbents tend to declare
   a high \( C \) for continuing the use of a block
   a low \( C \) for saving payment \( R \)
   tradeoff to incumbents
   “holding up” a block may be costly
5. Who should be “incumbents”?
   all users of spectrum
   private, business, and government users

D. Rights and responsibilities of potential users

We now turn to considering the behavior of new (potential) users of spectrum in the proposed EMM. In short, it is much the same as the one in the conventional market mechanism except that, in EMM, new users participate to the market through a centralized system operated by the government.

First of all, a new user would obtain information about supply prices \((C’s)\), as revealed by the incumbent users. If the new user found a block of spectrum that he/she would like to obtain, then he/she may make an offer with a demand price \((D)\) for the block chosen at a level greater than the supply price. If there is no competing offer, then the potential user would obtain the right to use the block for paying \(D\). If there are competing offer(s) by some other potential users, then auction will be conducted by the government on the block; a winning potential user would obtain the spectrum
right for paying the winning bid.

New Users in EMM:

1. Obtain information of $C$’s
2. Make offers by showing demand price ($D$) for blocks chosen
3. If there is no competing offer,
   then potential user obtains spectrum right for paying $D$.
4. If there is a competing offer,
   then auction will be conducted on such blocks
   winning potential user obtains spectrum right for paying winning bids.

E. Roles of government with EMM:

In this subsection, we summarize the roles of the government in EMM; in short, the government operates the market and manages the entire spectrum resources.

First of all, the government determines a fee rate ($r$), which will control the speed of reallocation of spectrum resources. In short, the fee rate works on the reallocation of spectrum resources in a way similar to that, in the financial world, the discount rate set by a central bank works on the nationwide allocation of investment funds. We will discuss this later in more detail.

Second, the government conducts auction on spectrum blocks for which multiple potential users have exhibited demand prices greater than the supply price. The role of the government in this regard would be similar to that of FCC of U.S., conducting auction on spectrum blocks for initial licensing, except that, in EMM, auctions would be conducted regularly (say, once a year) for reallocating spectrum blocks as designated by the market (that is, through revealed demand and supply prices). In the case of reallocation of multiple blocks with externalities, combinational auctions might be called for; the government should determine winning bids so as to maximize the total surplus from the spectrum transfer.

Third, the government will obtain revenues in two parts: the total spectrum usage fees paid by the incumbent users, and the total of the differences between winning bids and the supply prices (i.e., the total surplus) from the auctions conducted. The first may be regarded as the rental revenue to the society as a whole for having the incumbents use the spectrum resources, which are owned collectively. The second may be regarded as a “dividend” of improving the use of the spectrum.
resources.

Fourth, the government should conduct administrative task needed for EMM: registration of spectrum rights and their changes, registration and disclosure of revealed supply prices, bookkeeping of auction procedures, and disclosing information about all of these.

Roles of the Government in EMM:

1. spectrum holding fee
   a. determines a fee rate (r):
      to control the speed of reallocation
   b. receives spectrum fees (R)
2. market auctioneer
   a. conducts auction for each group with D > C
      use combinatorial auction (computerized)
      apply bidding rules, stopping rules
      determines winning bids so as to maximize the total amount of bid price minus C
      ( = total surplus)
   b. receives total surplus
3. collection and dissemination of information
   a. C, D, auction process, auction results
   b. the state of spectrum rights:
      registration
      information disclosure

F. Expected outcomes and welfare implications of EMM:

In this subsection, we discuss expected outcomes and welfare implications of EMM briefly.

First of all, we point out that EMM is a system for reallocating spectrum step by step. Thus, once EMM was implemented, then some of the spectrum blocks would be reallocated from old users to new users, so that the overall efficiency of spectrum use would be improved. If the economy were stationary with constant demand and no technological progress, we expect that EMM would eventually lead us to a Pareto-optimal allocation of the spectrum resources. In reality, however, the economy changes over time; as a consequence, we may expect that the “distance” between the actual
allocation of the spectrum resources and a Pareto-optimal one may only be “decreased,” not eliminated, by EMM. Below, we will show informally that EMM does have such capability.

We begin with considering welfare implications of the transfer of a spectrum block from an old user to a new user. We note that for the block being reallocated, it holds that the present value of the returns from the new use must be greater than the present value of the returns from the old use. This means that the block will produce more after the trade than before (see IIIE).

Next, we can confirm that the economic state of the old user will be better off after the trade than before, since the supply price of the block was so chosen that the incumbent will be better off if the block happened to be transferred. Likewise, the economic state of the new user will be better off after the trade, since, otherwise, he/she would not be engaged with the trade.

Thus, we can assert that, at the micro level, each trade in EMM will increase the efficiency of using the block traded, and also will improve the economic state of both the old and the new users (see IIIE.3).

We next turn to considering welfare implications of EMM at the macro level. See Figures VIF.1 and 2. In the figures, the horizontal axis measures the size of spectrum blocks, and the vertical axis the unit supply price of spectrum, which is equal to the amount of compensation divided by the size of a block. In Figure VIF.1, we express each spectrum block by a rectangle in the following way. First, the width of a rectangle is equal to the size of the block, and the area to the amount of compensation declared with that block. The height of the rectangle then expresses the unit supply price of the block. The rectangles are arranged from left to right in the increasing order of the unit supply prices. By combining the top of the rectangles, we obtain the “supply curve” of spectrum blocks, as shown in Figure VIF.2. Note that, in Figures VIF.1 and 2, we neglect the location of spectrum blocks on the surface of the earth and, instead, treat as if they were one-dimensional entity. This is merely for simplification.

Assume next that we can draw a demand curve from the demand prices revealed by the potential users as shown in Figure VIF.3. Note that there is no reason that such a demand curve is of downward-sloping. Suppose that the spectrum blocks traded for a time period (say, a year) under EMM is those corresponding to segment \( OA \) in the graphs. Then, the total amount paid by the new users is equal, in Figure VIF.3, to the area \( OADB \), which is divided into the total compensations received by the old users, area \( OAEC \), and the total surplus received by the government, area \( CEBD \).

Observe that EMM does not lead us to the optimal point, the intersection \( F \) of the demand and the supply curves; EMM can increase the efficiency of spectrum use only to a certain extent. We can expect, however, that the overall efficiency of spectrum use may be improved if EMM continues
to work for some time period (say, years). The demand and supply curves, however, will be shifted over time, and we cannot derive a definite assertion in this regard. Later, we will discuss about the “speed” of efficiency improvement with EMM in relation to the level of spectrum fee rate \(r\) chosen by the government (see VIIIE).

\(<\text{Figure VIF.1}>\)
\(<\text{Figure VIF.2}>\)
\(<\text{Figure VIF.3}>\)

VII. Applications and extensions of EMM

A. Commons users of spectrum

In this section, we examine the functioning of EMM for cases other than the case that all spectrum blocks are used exclusively as we assumed in the preceding sections. First, we consider how EMM works when a spectrum block is used as a club or as commons. In this case, a spectrum block is assigned to multiple users; it may be offered freely to the public as for the case of commons, or the entry by new users may be restricted by means of, say, licensing (as in amateur wireless) or by some other qualifications (as in a band used for the safety of navigation or aviation) as in the case of a club.\(^7\) For all of these cases, we expect that EMM works quite well. For simplicity, however, we consider in the following the case of commons only.

In order to apply the framework of EMM to commons, we will employ the following convention: the block (or the band) to be used as commons is first assigned exclusively to a public agent, to be called \textit{commons manager}, who then offers the block to the public for free use (but with some technical restrictions).

When EMM is implemented, commons (or club) users should understand and accept that the spectrum they are using may be taken away for reallocation. Each user can declare an amount of compensation which will be paid in the event of reallocation. Compensation may be declared by means of direct registration with the commons manager, or, if so chosen, by means of a declaration with a slip to be obtained at the time a device is purchased for using the commons block. For example, users of an electro-magnetic heater may wish to pay in one installment the compensation premiums for the period of expected duration of the equipment (e.g., for 10 years); the amount of

\(^7\) For the case of a club or commons, the distinction between allocation and assignment becomes unimportant, since a band is often composed of a single block, to which multiple users are assigned. We use, in such a case, the two terms of allocation and assignment interchangeably.
compensation may be set to reflect the purchase price of it, or, alternatively, the actual value of the equipment at the time of spectrum reallocation (e.g., 30% of the purchase price if the device becomes useless because of reallocation in the 7-th year of the 10-year duration period).

The total amount of compensations that the commons manager should pay for trading a commons block is equal to the sum of the declared compensations by all users of the block. Since spectrum commons is a public good, this is the supply-side analogue of the Lindahl-Samuelson valuation of public goods. In other words, the value of a commons block is expressed by the sum of the prices attached by all users of that block (See Figure VIIA.1).

To state the above formally, let $N$ represent the set of users of a commons block. A user is a member of this set, $n \in N$. We postulate that each user $n$ declares an amount $C_n$ as the compensation to be paid by the commons manager to the user in case the block is transferred to other user(s), and pays the compensation premium $rC_n$ to the commons manager annually, where $r$ is the rate of spectrum usage fee determined by the government, $n \in N$. The commons manager should declare the supply price $C$ equal to

$$C = \sum_{n \in N} C_n,$$

and pays the total spectrum fee

$$R = r \sum_{n \in N} C_n$$

annually to the government.

We note that the amount of compensation $C_n$ declared by user $n$ represents the value of commons to him/her. The sum $\sum C_n$ is equal to the value of the block (or the value of the services produced by the block) to the users.

Further, observe that, the commons manager selling the commons block may purchase a new block which can be offered to the users as a new commons with the level of service as great as with the commons sold and of which the cost is less than the total compensation; in this case, commons manager obtains a net surplus, which should be transferred to the government. In short, this is the case in which the value to the users of the services produced by the commons block is greater than the cost of supplying the services, i.e., the case in which the demand price is greater than the supply price so that a positive net surplus, the reallocation dividend of commons, can be realized.

In conclusion, we can state that EMM is expected to work well with club or commons assignment, of which reallocation is extremely difficult with the current system.
B. Subscribers to a service using spectrum

In many cases, spectrum is used directly and indirectly. Mobile operators supply mobile telephone services to subscribers by means of wireless technology; in this case, mobile operators are direct users and subscribers are indirect users; we call the latter end users. Likewise, TV stations use spectrum directly for broadcasting and consumers viewing TV are end users of spectrum. In this subsection, we consider how EMM works with end users. We call users of spectrum that are not end users such as mobile operators and broadcast stations intermediate users.

First of all, we point out that intermediate users of spectrum provide their services to end users usually in club or commons mode. Mobile telephone operators form, in effect, a club of mobile subscribers to have them share spectrum for mobile telephony. Consumers watch TV programs by using spectrum as commons which is provided by broadcast stations. Thus, it is possible to use the framework used for commons (clubs) when intermediate spectrum users deal with end users with regard to termination or modification of the service in the event of spectrum reallocation. This may be explained by means of examples as follows.

Consider intermediate spectrum users such as a mobile operator serving mobile telephone subscribers or a wireless Internet access provider serving Internet subscribers. Intermediate spectrum users (operators) can offer an EMM-like arrangement (contract) to their customers (mobile telephone or Internet subscribers). Then, the total amount of compensations to be paid by an operator to customers in the event of reallocation would be the sum of the compensations claimed by all customers. Therefore, the supply price that an operator should declare is the sum of the compensations to be paid to all customers for terminating the service plus the cost of reallocation incurred directly to the operator. Mobile or Internet subscribers may declare an amount of compensation and pay a premium at the time they begin subscription or they purchase devices for

Commons Users with EMM:

| primary user: | government administrator (commons manager) |
| secondary users: | general users (the public) |
| $C$: | the sum of all compensations declared by the users |
| $R$: | may be collected at purchasing a device for using a commons block |

(payment may be made together with that of insurance fees for breakage)
receiving a service.

For the case of free-to-air broadcasting, consumers use broadcast spectrum as commons; hence, EMM with commons as explained in the preceding subsection applies. For paid TV, consumers use broadcast spectrum in the club mode. An EMM-like contract may be formed between a broadcast station and end users of spectrum (consumers).

Thus, once EMM is introduced, then it will induce EMM-like arrangements between intermediate and end users of spectrum; we can expect that spectrum reallocation may be accomplished without serious troubles with end as well as intermediate users. In particular, if EMM had been implemented a couple of decades ago, then the entire project of digital-TV transition would have been an application of EMM to a reorganization of television channels, instead of being one of political and/or administrative hard workings.

Formally, let the end users be indexed by \( n = 1, \ldots, N \), where \( N \) is the total number of the end users. The end user \( n \) declares an amount \( C_n \) as the compensation to be paid by the operator in case the service contract is terminated because of reallocation of spectrum, and pays the compensation premium \( rC_n \) to the operator annually, where \( r \) is the fee rate of spectrum use to be determined by the government, \( n = 1, \ldots, N \). The operator, who works under EMM, is obliged to declare the supply price \( C = \sum_{n=1}^{N} C_n + C_0, \quad C_0 \geq 0 \), as the total compensation to be paid to him/her in case of reallocation, and pays the total spectrum fee \( R = r \sum_{n=1}^{N} C_n + rC_0 \) annually to the government, where \( C_0 \) stands for the net supply price that the operator receives in case of reallocation, and \( rC_0 \) is the net spectrum fee that the operator pays.

Figure VIIB.1 illustrates the relations among the government, the operator, and end users.
C. **Introduction of reallocation as a forward trading---forward supply prices**

This subsection is devoted to considering a form of EMM with multiple reallocation periods. 

*Reallocation period* is the time period between writing a contract for spectrum trade and its execution. In general, the cost of reallocation to the incumbent, to be included in the supply price in EMM, may greatly depend on the choice of a reallocation period. It is advantageous for both incumbent and potential users to introduce multiple reallocation periods so that the actual transfer of spectrum may be made with a reallocation period of minimum cost. The system introduced in this subsection is an EMM in which spectrum users can reveal their preferences over multiple reallocation periods.

Let us consider, as an example, a case with six different reallocation periods: 0-year period, 1-year period, …, 5-year period; the 0-year period corresponds to the case of single-reallocation period. Spectrum users are allowed to reveal the supply price for each of the six reallocation periods. It is convenient to organize the revelation of supply prices in the following way. First, the user reveals the supply price for reallocation with 5-year period. Next, the user reveals the supply price for decreasing the reallocation period by one year from 5 to 4 years. The supply price for reallocation with 4-year period is the sum of the 5-year price and the price for the one-year decrease. The supply price for each of 3, 2, 1, and 0-year periods are revealed similarly. See Figure VIIIC.1. The supply price for trade with 0-year period is, as stated above, equal to the one for the single reallocation period. Thus, this arrangement expands the domain of events on each of which a supply price is revealed without charging additional spectrum fees to be paid by users; i.e., this is a costless generalization of the system (aside from transactions cost).

Thus, spectrum users can exhibit their preferences over six different reallocation periods by
means of a supply price attached to each reallocation period. If it is convenient for an incumbent user to have, say, 3-year reallocation period (for the reason that, say, the average depreciation period of the user’s devices in use is 3 years), the supply price attached to the 3-year period will be far less than the price attached to the 2, 1, and 0-year periods.

<Figure VIIC.1>

D. Preventing speculations with EMM

One of the desirable properties of EMM is that it can prevent spectrum hold-up. As known from our experiences in reallocating land, it is possible for an owner of a space resource (such as land or spectrum) to attach an extremely high “supply price” for reallocation in an attempt to acquire extra profits. Holding-up a piece of space-type resource is quite effective when it is located in the middle of a large-sized space which is about to be reallocated. This is an outcome of positive externalities in using land or spectrum resources. The social cost arising from holding-up may be very high, since it may prevent users from efficient trade of spectrum.

A typical case for a spectrum user to reveal a supply price speculatively for making extra profits may be like the following. Such a user would seek a spectrum block located strategically with regard to the supply prices revealed by other users. When the supply prices revealed are relatively low for blocks located near to a strategic one, then because of positive externalities, the probability that the strategic block is obtained by a potential user is high even if it carries a relatively high supply price. A speculative user would seek such profit-making opportunities, which will be a factor disturbing smooth functioning of EMM.

Figures VIID.1 and 2 illustrate such a speculation, where the horizontal axis measures the quantity of spectrum blocks, and the vertical axis the unit price of spectrum. In Figure VIID.1, five blocks of different sizes are depicted, to each of which truthful unit supply price is attached. Suppose that there are positive externalities between these blocks, which may be traded soon. Suppose further that the third block is of small size relative to the others; it can then be a strategic block. The user of this block may speculatively reveal a very high false supply price, as shown in Figure VIID.2. If the potential user makes a decision solely on the total amount of supply prices, these five blocks may still be traded, since the increase in the payment for the blocks due to the speculation may not be large; the speculation will succeed.

To avoid such speculations and accompanying disturbances, the government may impose the
following regulations. First, the government should allow incumbent users to revise the supply prices from time to time. Further, the government should disclose all information about the supply prices revealed including their changes in the past. If there is a speculative pricing with sharp increases during recent periods, then the potential buyer may be warned of such speculations by examining the record of supply prices in the past. In this way, speculative pricing may be eliminated gradually through competition among incumbent users, although complete elimination of speculation may not be possible.

<Figure VIID.1>

<Figure VIID.2>

E. Transition from the current system to EMM

It is noted that EMM is a system with which the speed of implementation may be controlled by the government. It would be desirable to implement EMM slowly and gradually, rather than to introduce it big-bang, in order to avoid giving excessive economic shocks to incumbents. The following is a way to do this.

In the beginning, EMM may be introduced with an extremely low fee rate \( r \), perhaps a near-zero level. Spectrum users may reveal a very high supply prices, but pay almost nil. Then, the level of spectrum trade will be very low. During this period, a database may be constructed to store and publish information about the supply prices revealed together with their statistics. In this way, both incumbent and potential users can learn about the average level of current supply prices of spectrum, if distorted because of a very low rate of premium.

After a few years, the government may start raising the fee rate gradually. Spectrum trade may begin with blocks of which the difference between the demand and the supply prices is large. Spectrum users will start feeling the burden of paying spectrum usage fees, and will adjust their supply prices accordingly. It is expected that, as time goes on, incumbent users learn more and adjust their supply prices toward the level of truthful ones.

Figures VIIE.1 and 2 illustrate such a process of gradual implementation of EMM, where \( t = t_0 \) is the time at which EMM is first introduced. Figure VIIE.1 is a graph of the time path of the fee rate, which approaches to a (long-run) optimal level \( r^* \). Figure VIIE.2 illustrates the process in which the total fee revenue on the current system is replaced gradually by the fee revenue from

\[ \text{Note that such a database may be built on top of the database used for storing information of individual licenses currently.} \]
EMM, where it is assumed that the spectrum users are allowed to deduct their fee payment on EMM from that on the current system.

<Figure VII.E.1>

<Figure VII.E.2>
References


Figure IIB.1: Examples of Spectrum Band in the Frequencies Axis

Figure IIB.2: Example of Spectrum Block(B, A) in the 3-dimentional Spectrum Space
Figure IIB.3: Example of 10 Spectrum Blocks

Figure IIB.4: 2 Bands and 6 Areas for the Blocks of Figure 3
**Figure IIIB.1:** Business resources of X with and without the block B

**Figure IIIB.2:** Value of X with and without B and the supply price of B by X
**Figure IIIC.1:** Business resources of Y without and with the block B

- A: Without the block
- B: With the block

**Figure IIIC.2:** Value of Y without and with B and the demand price for B by Y

- A: Without the block
- B: With the block
Figure IVC.1: Spectrum Groups (Block Structure)

AB (15)

A (5)  B (5)

Figure IVC.2: Valuation of blocks A, B and group AB

ABC (25)

AB (15)

A (5)  B (5)  C (5)

Figure IVC.3: Valuation of blocks A, B, C and groups AB, ABC
Figure VD.1: Trade prices of block B
Figure VE.1: Possible pricing of block B for trade as seen by X
Figure VE.2: Possible pricing of block B for trade as seen by Y.
Figure VF.1: Example of successful bargaining by X and Y of block B
Figure VG.1: Unsuccessful bargaining by X and Y of block B---Typical case
Figure VH.1: Introduction of conventional market for spectrum trade in a stationary economy
Figure VH.2: Introduction of conventional market for spectrum trade
With technological progress
Figure VIA.1: Division of public regulation of spectrum into two sections: (A) and (B)

(A) Engineering/Technological Regulation

- Spectrum division (bands, blocks)
- Objectives and specifications of spectrum use
- Technological requirements
- Introduction of block structure

(B) Extended Market Mechanism (EMM)

- Users: participate in spectrum trade
- Government: operation of spectrum market
- Disclosure of spectrum information

Figure VIA.2: Organization of EMM

Demand side (Potential Users, Y)

1. Revelation of demand price \( (D=P_Y^*) \)
2. Participate in auction
3. If wins, obtain spectrum and become a user

Supply side (Incumbents, X)

1. Revelation of supply price \( (C=P_X^*) \)
2. Yield spectrum right if an offer \( > C \) is made
   - Receive compensation \( (C) \)
3. Pay spectrum holding fee \( (R) \) \( (R = rC) \)

Market regulator (Government, Z)

1. Specify spectrum-holding fee rate \( (r) \)
2. Receive spectrum-holding fees \( (R) \)
3. Execute auction if excess demand arises
4. Receive auction surplus
Figure VIF.1: “Supply” of Spectrum Blocks (1/2)

Figure VIF.2: “Supply” of Spectrum Blocks (2/2)
Figure VIF.3: Spectrum trade with EMM expressed by means of “Demand and Supply Curves”
Figure VIIA.1: Supply Price Revealed by Commons Users

- Government
  - Fee Rate ($r$)
  - Fees Paid ($rc$)
  - Total Amount of Compensations Declared ($C$)

- Commons Manager
  - Fee Rate ($r$)
  - Fees Paid ($rc$)
  - Amount of Compensation in case of Spectrum Reallocation ($c$)

- Commons Users
  - Fee Rate ($r$)
  - Fees Paid ($rc$)
Figure VII.B.1: Supply Prices Revealed by a Service Provider and Subscribers
Note: A shaded area denotes the increase in the supply price when the period of trade execution is shortened by 1 year.

**Figure VIIC.1:** Supply Prices in Forward Trading of Spectrum
Figure VIID.1: Example of truthful supply prices
Figure VIID.2: Examples of truthful and untruthful supply prices
Figure VII.E.1: Proposed time path of spectrum–fee rate

Figure VII.E.2: Expected change of spectrum-fee revenues